

Gauge-invariance properties and singularity cancellations in a modified PQCD

A. Cabo^{1,2,a}, M. Rigol³

¹ Theory Division, CERN, Geneva, Switzerland

² Group of Theoretical Physics, Instituto de Cibernética, Matemática y Física, calle E, No. 309, Vedado, la Habana, Cuba

³ Physics Department, University of California, Davis, CA 95616, USA

Received: 4 November 2005 /

Published online: 15 May 2006 – © Springer-Verlag / Società Italiana di Fisica 2006

Abstract. The gauge-invariance properties and singularity elimination of the modified perturbation theory for QCD introduced in previous works are investigated. The construction of the modified free propagators is generalized to include the dependence on the gauge parameter α . Further, a functional proof of the independence of the theory under the changes of the quantum and classical gauges is given. The singularities appearing in the perturbative expansion are eliminated by properly combining dimensional regularization with the Nakanishi infrared regularization for the invariant functions in the operator quantization of the α -dependent gauge theory. First-order evaluations of various quantities are presented, illustrating the gauge-invariance properties.

PACS. 12.38.Aw; 12.38.Bx; 12.38.Cy; 14.65.Ha

1 Introduction

Improving the understanding of chromodynamics is an important issue in modern particle physics [1]. Although the small couplings at high momenta allow for the use of perturbation theory in this region, the expansion is however unable to describe the physics at low energies. In former works [2–6] we have been considering the development of a modified perturbation expansion, searching to describe at least some low-energy properties. The proposed scheme [2] conserves color and Lorentz invariance. Indeed, it was motivated by the aim of eliminating the symmetry limitations of the earlier chromomagnetic field models [7–10]. The Feynman expansion proposed in [2] included in the gluon propagator a term produced by the condensation of zero-momentum gluons. Later, in [6, 11] the interest of also including quark condensates was argued. Just in the first approximation, the modified rules produced a non-vanishing gluon condensation parameter $\langle g^2 G^2 \rangle$ [12]. Afterwards, in [2], a non-zero tachyon mass for the gluons was evaluated at the one-loop level. This outcome was consistent with the conclusion in [13] about the tachyon character of the normal Green functions in the presence of a gluon condensate [14, 15]. Also, the one-loop effective potential as a function of the condensate parameter indicated its dynamic generation. The plausibility of modifying the Feynman rules of QCD, including gluon condensation, was also foreseen in [16]. In this work a filling

of gluon states for all momenta $k < \Lambda_{\text{qcd}}$, was assumed in defining an alternative free vacuum state, upon which to connect the color interaction.

The addition of a delta-function term at zero momentum in the gluon propagator had also been investigated in [17, 18]. The aim was to develop a phenomenological model for meson resonances, showing a confining interquark potential. The multiplicative constant of the delta function had the opposite sign with respect to the one chosen in [3, 5]. For this selection of the sign, the singularities of the quark propagator showed no pole on the real p^2 axis [18]. Thus, the results led to a physical description different from that in [2–5].

The expansion examined in [2] was derived in [3, 5] as the result of the adiabatic connection of the color interaction, upon a particularly constructed free physical state. The operator quantization scheme of Kugo and Ojima [19, 20] allowed this state to be determined by the creation of zero-momenta gluons and ghost particles. The propagators considered in [2] then emerged as generating the Wick expansion. The annihilation of the modified vacuum by the action of the Becchi–Rouet–Stora–Tyutin (BRST) charge assured its physical character at zero coupling. It also followed that the parameter describing the condensation should be real and positive. This specific definition, in our case, discarded the possibility of fixing the sign of the factor multiplying the delta function, in the way chosen in [17, 18]. It should be added that the propagators employed by Munczek and Nemirovsky were also obtained in [21], as generated by the Wick expansion from a vacuum

^a e-mail: cabo@cidet.icmf.inf.cu, Alejandro.Cabo@cern.ch

including condensed gluons in a squeezed state. It may be that the BRST physical condition also could be satisfied by that class of states. Then, their alternative approach could be constructed along the same lines. The difference in the end results for these two treatments might be related with the fact that two different kinds of “squeezed” gluon states were employed [21].

In the present work, we consider the investigation of the gauge invariance and regularization properties of the modified expansion under study. Our main objectives will be:

- (a) to generalize the procedure to arbitrary values of the gauge parameter α
- (b) to eliminate the infrared singularities appearing as a consequence of the delta-function structure of the new propagators
- (c) to present the formal proof of the Ward–Takahashi–Slavnov identities and the gauge-parameter independence of the physical quantities, and finally
- (d) to check the satisfaction of the gauge independence of the effective action and the transversality of the gluon self-energy at zero values of the mean fields, to second order in the coupling g and all orders in the expansion in the condensate parameters.

Section 2 starts by presenting the initial state to be used in the Wick expansion formula. It is then employed in the derivation of the modified propagators. At this point, the invariant functions appearing as the results of the commutators defining all the propagators are modified by implementing the Nakanishi procedure for the infrared regularization of these functions [22]. This procedure allows us to show that all the Feynman parts of the propagators are regularized to be equal to zero in a small neighborhood of the origin in momentum space. The size of the vanishing region is fixed by the Nakanishi regularization parameter σ . This is a basic outcome that further allows the elimination of a large class of the singularities generated by the delta-function character of the modified propagators.

Section 3 is devoted to deriving the Ward–Takahashi–Slavnov (WTS) identities corresponding to the Green functions in the modified expansion [23–25]. They fully coincide with those in the usual theory (PQCD). This outcome is suggested after considering that the modified propagators are allowed inverse kernels of the tree-level inverse propagators of massless QCD. The generating functional of the Green functions is transformed into a Feynman integral form, only differing in the boundary conditions on the fields, from the standard one in PQCD.

In Sect. 4, we formulate a procedure for the elimination of the infrared singularities appearing in any order of the perturbative expansion due to the $\delta(p)$ form of the condensate terms. These divergences can be cancelled in two ways. The first one is using the dimensional regularization for the Dirac’s delta function evaluated at zero momentum. As was shown in [26], in the case of spatial arguments, these expressions can be made to vanish in dimensional regularization. An identical proof is employed here to argue the same conclusion for momentum-space arguments. Using this rule, a large class of the singular terms are removed. The second recourse leading to a vanishing result

for the remaining singular terms is the Nakanishi infrared regularization described in Sect. 2. Specifically, the equality to zero of the Feynman contribution to the gluon, quark and ghost propagators in the neighborhood of $p = 0$ (of a size fixed by the Nakanishi parameter σ) allows us to cancel the remaining singularities. Since these singular terms arise from the evaluation of the Feynman propagators at zero momenta, this is simply obtained after setting the absolute values of the momenta at which the condensate states are created, being sufficiently smaller than σ .

Then, Sect. 6 presents the checking of the transversality of the polarization tensor and the gauge-parameter independence of the effective action evaluated at zero value of the mean fields. This is done here up to second order in the coupling constant and any order in the condensate parameters. Afterwards, comments on the results and possible directions of their future extension are given in a summary.

Finally a resume of the Kugo–Ojima quantization procedure for free massless QCD for general value of the gauge parameter α , and a complementary calculation, are given in Appendices A and B, respectively.

2 Modified Feynman expansion

2.1 The initial vacuum for the Wick series

Let us consider in what follows the selection of the initial vacuum state of free QCD upon which the interaction will be connected. It will be a generalization of the previously defined one in [3, 5], but physically equivalent to it, since its physical part (the transverse gluons part) is not modified. The changes appear only in the non-physical sector, which we choose the most general possible in order to have a wider freedom in the selection of the resulting propagators. The main elements of the Kugo–Ojima operator quantization of the free massless QCD for an arbitrary gauge parameter α are reviewed in Appendix A. The resume collects the equations of motion and commutation relations between the fields and the creation and annihilation operators. The definitions of the invariant functions and the various sorts of wave packets to be employed below can also be found there. The vacuum state will be defined in the form

$$\begin{aligned}
 |\Psi\rangle = \exp & \left\{ \sum_{a=1}^8 \sum_{\mathbf{p}_i, |\mathbf{p}_i|=P<\sigma} \left[\sum_{\lambda=1,2} \frac{C_\lambda(P)}{2} A_{\mathbf{p}_i, \lambda}^{a+} A_{\mathbf{p}_i, \lambda}^{a+} \right. \right. \\
 & + C_3(\alpha, P) (B_{\mathbf{p}_i}^{a+} A_{\mathbf{p}_i}^{L, a+} + i\bar{c}_{\mathbf{p}_i}^{a+} c_{\mathbf{p}_i}^{a+}) \\
 & \left. \left. + D(\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+} \right] \right. \\
 & \left. + \sum_{s=1,2} \sum_{\mathbf{p}_i, \mathbf{q}_i, |\mathbf{p}_i, \mathbf{q}_i|=P<\sigma} C_{\mathbf{p}_i, \mathbf{q}_i} b_{\mathbf{q}_i}^{s+} a_{\mathbf{p}_i}^{s+} \right\} |0\rangle. \tag{1}
 \end{aligned}$$

The gluon (first part of the exponential) and quark (second part of the exponential) pairs defining the state are

created for a set of small but non-vanishing momenta \mathbf{p}_i in order to avoid the difficulties occurring if they are directly chosen to have zero momentum. A sum over the set \mathbf{p}_i for $|\mathbf{p}_i| = P < \sigma$ is introduced, to have freedom in eliminating any preferential direction in the space, remaining after the Nakanishi parameter is taken in the limit $\sigma \rightarrow 0$.

In order to simplify the exposition, we evaluate gluon expressions below for a generic mode of given color a and momentum \mathbf{p}_i indices. This can be done because for the free theory, and for the specific quantities to be evaluated, contributions of different modes can be worked out independently, thanks to the commutation relations (A.6). At the necessary point in the discussion, all contributions will be included.

The proof that the proposed state (1) satisfies the required BRST physical-state conditions,

$$Q_B |\Psi\rangle = i \sum_{\mathbf{k}, a} (c_{\mathbf{k}}^{a+} B_{\mathbf{k}}^a - B_{\mathbf{k}}^{a+} c_{\mathbf{k}}^a) |\Psi\rangle = 0,$$

$$Q_C |\Psi\rangle = i \sum_{\mathbf{k}, a} (\bar{c}_{\mathbf{k}}^{a+} c_{\mathbf{k}}^a + c_{\mathbf{k}}^{a+} \bar{c}_{\mathbf{k}}^a) |\Psi\rangle = 0,$$

proceeds as follows.

Using the commutation relations (A.6), the state $|\Psi\rangle$ in (1) can be written in the form

$$|\Psi\rangle = \exp(D(\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+}) |\Phi\rangle,$$

where $|\Phi\rangle$ has the form

$$|\Phi\rangle = \exp \left[C_1(P) A_{\mathbf{p}_i, 1}^{a+} A_{\mathbf{p}_i, 1}^{a+} + C_2(P) A_{\mathbf{p}_i, 2}^{a+} A_{\mathbf{p}_i, 2}^{a+} + C_3(\alpha, P) (B_{\mathbf{p}_i}^{a+} A_{\mathbf{p}_i}^{L, a+} + i \bar{c}_{\mathbf{p}_i}^{a+} c_{\mathbf{p}_i}^{a+}) \right] |0\rangle$$

and satisfies the physical-state conditions, as was probed in [3, 5],

$$Q_B |\Phi\rangle = Q_C |\Phi\rangle = 0.$$

The commutation relations (A.6) also allow one to show that

$$\begin{aligned} [Q_B, \exp(D(\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+})] &= 0, \\ [Q_C, \exp(D(\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+})] &= 0. \end{aligned}$$

Hence

$$\begin{aligned} Q_B |\Psi\rangle &= \exp(D(\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+}) Q_B |\Phi\rangle = 0, \\ Q_C |\Psi\rangle &= \exp(D(\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+}) Q_C |\Phi\rangle = 0. \end{aligned}$$

The state (1) is then a physical state of the free theory. The non-physical sector of $|\Psi\rangle$ is undetectable in the physical world and can be neglected when calculating physical quantities like the norm, energy, and particle number.

As stated before, the physical sector of (1) is the same as proposed in previous work [3, 5]; then for the calculation of the norm a similar result is obtained:

$$\begin{aligned} N &= \langle \Psi | \Psi \rangle \\ &= \prod_{\lambda=1, 2} \prod_{a=1}^8 \prod_{\mathbf{p}_i, |\mathbf{p}_i|=P < \sigma} \left[\sum_{m=0}^{\infty} |C_{\lambda}(P)|^{2m} \frac{(2m)!}{(m!)^2} \right], \\ &\text{for } |C_{\lambda}(P)| < 1. \end{aligned}$$

The normalized physical vacuum state is defined as

$$|\tilde{\Psi}\rangle = \frac{1}{\sqrt{N}} |\Psi\rangle. \quad (2)$$

The modifications introduced in the usual perturbation theory by the state (1) are calculated in this section for arbitrary values of the gauge parameter α . For this purpose the expressions derived in [5, 28] for the Green function generating functional of a free theory are used

$$Z_0[j, \xi, \bar{\xi}, \eta, \bar{\eta}] = Z_0^g[j] Z_0^q[\eta, \bar{\eta}] Z_0^{gh}[\xi, \bar{\xi}], \quad (3)$$

$$\begin{aligned} Z_0^g[j] &= \langle \tilde{\Psi} | T \exp \left(i \int dx j^{\mu}(x) A_{\mu}(x) \right) | \tilde{\Psi} \rangle \\ &= \langle \tilde{\Psi} | \exp \left(i \int dx j^{\mu}(x) A_{\mu}^{-}(x) \right) \\ &\quad \times \exp \left(i \int dy j^{\mu}(x) A_{\mu}^{+}(x) \right) | \tilde{\Psi} \rangle \\ &\quad \times \exp \left(\int dx dy \theta(y_0 - x_0) j^{\mu}(x) \right. \\ &\quad \left. \times [A_{\mu}^{-}(x), A_{\nu}^{+}(y)] j^{\nu}(y) \right), \quad (4) \\ &= Z_0^{g,m}[j] Z_0^{g,F}[j], \quad (5) \end{aligned}$$

$$\begin{aligned} Z_0^q[\eta, \bar{\eta}] &= \langle \tilde{\Psi} | \exp \left(i \int dx \right. \\ &\quad \left. \times [\bar{\eta}(x) \psi^{-}(x) + \bar{\psi}^{-}(x) \eta(x)] \right) \\ &\quad \times \exp \left(i \int dy [\bar{\eta}(x) \psi^{+}(x) + \bar{\psi}^{+}(x) \eta(x)] \right) \\ &\quad \times | \tilde{\Psi} \rangle \exp \left(\int dx dy \bar{\eta}(y) [\theta(x_0 - y_0) \right. \\ &\quad \left. \times \{ \psi^{-}(y), \bar{\psi}^{+}(x) \} - \theta(y_0 - x_0) \right. \\ &\quad \left. \times \{ \psi^{+}(y), \bar{\psi}^{-}(x) \} \right] \eta(x) \Big), \quad (6) \\ &= Z_0^{q,m}[\eta, \bar{\eta}] Z_0^{q,F}[\eta, \bar{\eta}], \quad (7) \end{aligned}$$

$$\begin{aligned} Z_0^{gh}[\xi, \bar{\xi}] &= \langle \tilde{\Psi} | \exp \left(i \int dx \right. \\ &\quad \left. \times [\bar{\xi}(x) c^{-}(x) + \bar{c}^{-}(x) \xi(x)] \right) \\ &\quad \times \exp \left(i \int dy [\bar{\xi}(x) c^{+}(x) + \bar{c}^{+}(x) \xi(x)] \right) \\ &\quad \times | \tilde{\Psi} \rangle \exp \left(\int dx dy \bar{\xi}(y) [\theta(x_0 - y_0) \right. \\ &\quad \left. \times \{ c^{-}(y), \bar{c}^{+}(x) \} - \theta(y_0 - x_0) \right. \\ &\quad \left. \times \{ c^{+}(y), \bar{c}^{-}(x) \} \right] \xi(x) \Big), \quad (8) \\ &= Z_0^{gh,m}[\xi, \bar{\xi}] Z_0^{gh,F}[\xi, \bar{\xi}], \quad (9) \end{aligned}$$

where the color and spinor indices are implicit. As is clear from the above formulae, the changes in the gluon, quark

and ghost generating functionals with respect to the standard ones in PQCD are given by the factors

$$\begin{aligned}
Z_0^{g,m}[j] &= \langle \tilde{\Psi} | \exp \left(i \int dx j^\mu(x) A_\mu^-(x) \right) \\
&\quad \times \exp \left(i \int dy j^\mu(x) A_\mu^+(x) \right) | \tilde{\Psi} \rangle, \\
Z_0^{q,m}[\eta, \bar{\eta}] &= \langle \tilde{\Psi} | \exp \left(i \int dx [\bar{\eta}(x) \psi^-(x) + \bar{\psi}^-(x) \eta(x)] \right) \\
&\quad \times \exp \left(i \int dy [\bar{\eta}(x) \psi^+(x) + \bar{\psi}^+(x) \eta(x)] \right) \\
&\quad \times | \tilde{\Psi} \rangle, \\
Z_0^{gh,m}[\xi, \bar{\xi}] &= \langle \tilde{\Psi} | \exp \left(i \int dx [\bar{\xi}(x) c^-(x) + \bar{c}^-(x) \xi(x)] \right) \\
&\quad \times \exp \left(i \int dy [\bar{\xi}(x) c^+(x) + \bar{c}^+(x) \xi(x)] \right) \\
&\quad \times | \tilde{\Psi} \rangle, \tag{10}
\end{aligned}$$

which all reduce to unity in the case of the standard PQCD.

Let us now explicitly state the infrared regularization rules of the expansion to be considered.

- (a) The invariant functions D and E , in terms of which the commutators (anti-commutators) defining the standard Feynman propagators are expressed, will be assumed to be given by the Nakanishi infrared regularized expressions

$$\begin{aligned}
D^{(\pm)}(x-y|\sigma, m) &= \pm \int \frac{dk}{(2\pi)^3} \theta(\pm k_0 - \sigma) \delta(k^2 - m^2) \\
&\quad \times \exp(-ik(x-y)), \\
E^{(\pm)}(x-y|\sigma) &= \frac{1}{2} \frac{1}{\nabla_{(x-y)}^2} \left[(x_0 - y_0) \frac{\partial}{\partial(x_0 - y_0)} - 1 \right] \\
&\quad \times \lim_{m \rightarrow 0} D^{(\pm)}(x-y|\sigma, m). \tag{11}
\end{aligned}$$

- (b) For the evaluation of the changes in the propagators produced by the condensation, which are physically related to the zero-momentum modes of the theory, the unmodified field operators will be used. In addition, the spatial momenta for which gluon and quark condensate states are created will be assumed to lie well within a small neighborhood of $\mathbf{p} = 0$. The radius P of this region will be chosen much smaller than the Nakanishi parameter σ in (11).

Let us consider in what follows the evaluation of the various Feynman propagators following the above prescriptions.

2.2 Regularized Feynman propagators

2.2.1 Gluons

The Feynman propagator for the gluons is determined by the commutator between the creation and annihilation

parts of their field operators. After substituting (A.4) in this commutator, the expression can be transformed in the following way:

$$\begin{aligned}
[A_\mu^{a-}(x), A_\nu^{b+}(y)] &= -\delta^{ab} g_{\mu\nu} D_+(x-y|0) - (1-\alpha) \delta^{ab} \\
&\quad \times \left(\partial_\nu^y D^{(\frac{1}{2})}(y) \sum_{\mathbf{k}} g_k^*(y) f_{k,L,\mu}(x) \right. \\
&\quad \left. + \partial_\nu^x D^{(\frac{1}{2})}(x) \sum_{\mathbf{k}} g_k(x) f_{k,L,\mu}^*(y) \right) \\
&= \lim_{\sigma \rightarrow 0} \left(-\delta^{ab} [g_{\mu\nu} D_+(x-y|\sigma, 0) \right. \\
&\quad \left. - (1-\alpha) \partial_\nu^y \partial_\mu^x E_+(x-y|\sigma) \right], \tag{12}
\end{aligned}$$

where we have defined $D_\pm(x-y|0) = \lim_{\sigma \rightarrow 0} D_\pm(x-y|\sigma, 0)$. In the last line of this relation, in which the commutator has been expressed in terms of the invariant functions, the Nakanishi regularization (11) has been introduced. The functions taken for a finite value of σ will be employed in what follows, defining in this way the regularized propagators.

Substituting the above formula in the quadratic form in the gluon sources defining the Feynman contribution $Z_0^{g,F}$ for the generating functional in (14), we now have the form

$$\begin{aligned}
S_T &= \int dx dy \theta(y_0 - x_0) j^\mu(x) [A_\mu^-(x), A_\nu^+(y)] j^\nu(y) \\
&= \frac{1}{2} \int dx dy j^{\mu,a}(x) g_{\mu\nu} [\theta(y_0 - x_0) D_+(x-y|\sigma, 0) \\
&\quad + \theta(x_0 - y_0) D_+(y-x|\sigma, 0)] j^{\nu,a}(y) \\
&\quad + \frac{(1-\alpha)}{2} \int dx dy \partial_\mu^x j^{\mu,a}(x) [\theta(y_0 - x_0) E_+(x-y|\sigma) \\
&\quad + \theta(x_0 - y_0) E_+(y-x|\sigma)] \partial_\nu^y j^{\nu,a}(y) \\
&= \frac{1}{2} \int dx dy j^{\mu,a}(x) g_{\mu\nu} D_F(x-y|\sigma) j^{\nu,a}(y) \\
&\quad + \frac{(1-\alpha)}{2} \int dx dy \partial_\mu^x j^{\mu,a}(x) E_F(x-y|\sigma) \partial_\nu^y j^{\nu,a}(y) \\
&= S_D + S_E, \tag{13}
\end{aligned}$$

where, in order to shift the derivatives to act on the sources, the following properties have been used:

$$\begin{aligned}
\partial_\nu^y \partial_\mu^x [E_F(x-y|\sigma)] &= \partial_\nu^y \partial_\mu^x [\theta(x_0 - y_0) E_+(x-y|\sigma) \\
&\quad - \theta(y_0 - x_0) E_-(x-y|\sigma)] \\
&= [\theta(x_0 - y_0) \partial_\nu^y \partial_\mu^x E_+(x-y|\sigma) \\
&\quad - \theta(y_0 - x_0) \partial_\nu^y \partial_\mu^x E_-(x-y|\sigma)],
\end{aligned}$$

in which we have defined $\partial_\mu = \nabla_\mu + u_\mu \partial_0$ with $\nabla_\mu \equiv (0, \partial_1, \partial_2, \partial_3)$, $u_\mu \equiv (1, 0, 0, 0)$, and the final equality follows because the delta functions evaluate the factors $(E_+(x-y) + E_-(x-y))$ at $x_0 = y_0$, where $E_+(x-y|\sigma) = -E_-(x-y|\sigma)$ is satisfied. Then the Feynman propagator contribution to the generating functional of the gluons takes the

form

$$\begin{aligned} \exp(S_D + S_E) &= \exp \left(\int dx dy \theta(y_0 - x_0) j(x) \right. \\ &\quad \left. \times [A^-(x), A^+(y)] j(y) \right) \quad (14) \\ &= \exp \left[\frac{1}{2} \int dx dy j^{\mu,a}(x) g_{\mu\nu} D_F(x-y|\sigma) \right. \\ &\quad \times j^{\nu,a}(y) + \frac{(1-\alpha)}{2} \int dx dy \partial_\mu^x j^{\mu,a}(x) \\ &\quad \left. \times E_F(x-y|\sigma) \partial_\nu^y j^{\nu,a}(y) \right], \end{aligned}$$

where $D_F(x-y|\sigma)$ indicates the Feynman term in the modified gluon propagator for $\alpha = 1$, although it differs by a constant multiplicative factor from the usual definition.

The generating functional expressed in momentum space for the gluons can be obtained now by passing the quadratic integrals on the sources to the Fourier representation. The following convention for the Fourier transform of any quantity Q and the Fourier transform of the Heaviside function will be used:

$$\begin{aligned} Q(x) &= \int \frac{dq}{(2\pi)^4} Q(q) \exp(-iqx) \\ &\equiv \int \frac{dq}{(2\pi)^4} \mathcal{FT}(Q(x)|x, q) \exp(-iqx), \quad (15) \end{aligned}$$

$$\theta(x_0) = \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \frac{i}{q_0 + i\varepsilon} \exp(-iq_0 x_0). \quad (16)$$

Let us consider first the second term in the argument of the exponential of S_E , which is the most involved one. After substituting the Fourier representations (15) for the sources, the Heaviside function (16) and (A.3), we can obtain

$$\begin{aligned} S_E &= -\frac{(1-\alpha)}{2} \int \frac{dk}{(2\pi)^4} k^\mu j_\mu^a(-k) [\mathcal{FT}(\theta(z_0)E_+(z|\sigma)|z, k) \\ &\quad + \mathcal{FT}(\theta(z_0)E_+(z|\sigma)|z, -k)] k^\nu j_\nu^a(k). \quad (17) \end{aligned}$$

The first Fourier transform in the integrand of (17), after substituting the definition (11) of the E function and the Fourier representation (16) of $\theta(z_0)$, can be obtained in the form

$$\mathcal{FT}(\theta(z_0)E_+(z|\sigma)|z, k) = \frac{i\theta(|\vec{k}| - \sigma)(2|\vec{k}| - k_0)}{-4|\vec{k}|^3(k_0 - |\vec{k}| + i\varepsilon)^2}.$$

Since the second Fourier transform in the integrand of (17) is simply the first one, but with the integration momentum k argument replaced by $-k$, adding the two terms

leads to

$$\begin{aligned} S_E &= \frac{(1-\alpha)}{2} \int \frac{dk}{(2\pi)^4} k^\mu j_\mu^a(-k) \left[\frac{i\theta(|\vec{k}| - \sigma)(2|\vec{k}| - k_0)}{-4|\vec{k}|^3(k_0 - |\vec{k}| + i\varepsilon)^2} \right. \\ &\quad \left. + \frac{i\theta(|\vec{k}| - \sigma)(2|\vec{k}| + k_0)}{-4|\vec{k}|^3(-k_0 - |\vec{k}| + i\varepsilon)^2} \right] k^\nu j_\nu^a(k) \quad (18) \end{aligned}$$

$$= -\frac{(1-\alpha)}{2} \int \frac{dk}{(2\pi)^4} k^\mu j_\mu^a(-k) \frac{i\theta(|\vec{k}| - \sigma)}{(k^2 + i\varepsilon)^2} k^\nu j_\nu^a(k). \quad (19)$$

For the terms in (14) associated to the invariant function D , a similar but much simpler calculation, thanks to the absence of dipole fields, allows us to obtain

$$S_D = \int \frac{dk}{(2\pi)^4} j_\mu^a(-k) \frac{i\theta(|\mathbf{k}| - \sigma)}{2(k^2 + i\varepsilon)} g^{\mu\nu} j_\nu^a(k),$$

which, inserted in (14) gives for the regularized gluon Feynman factor in the free generating functional the expression

$$\begin{aligned} Z_0^{g,F}[j] &= \exp \left(\frac{i}{2} \int \frac{dk}{(2\pi)^4} j^{\mu,a}(-k) \frac{i\theta(|\mathbf{k}| - \sigma)}{(k^2 + i\varepsilon)} \right. \\ &\quad \left. \times \left[g_{\mu\nu} - \frac{(1-\alpha)k_\mu k_\nu}{(k^2 + i\varepsilon)} \right] j^{\nu,a}(k) \right). \end{aligned}$$

2.2.2 Ghosts and quarks

In a similar way, the regularized ghost and quark propagators (the quark taken in the massless limit $m \rightarrow 0$) can be obtained in the forms

$$Z_0^{g,F}[\eta, \bar{\eta}] = \exp \left(i \int \frac{dk}{(2\pi)^4} \bar{\eta}(-k) \frac{-\theta(|\vec{k}| - \sigma) k_\mu \gamma^\mu}{k^2 + i\varepsilon} \eta(k) \right), \quad (20)$$

$$Z_0^{gh,F}[\xi, \bar{\xi}] = \exp \left(i \int \frac{dk}{(2\pi)^4} \bar{\xi}(-k) \frac{-i\theta(|\vec{k}| - \sigma) \delta^{ab}}{k^2 + i\varepsilon} \xi(k) \right). \quad (21)$$

Therefore, the Nakanishi regularization leads to the vanishing of all the Feynman propagator terms within a tube containing $k = 0$ and defined by the momenta having modulus of their spatial component smaller than σ , for arbitrary values of the temporal component k_0 . In this way, as will be seen further in the discussion, the adoption of this regularization procedure naturally leads to the cancelation of a great number of the singularities in the perturbative expansion.

2.3 Condensation-induced changes in the propagators

The effects on the propagators of the gluons and quarks created by the condensation of gluons, quarks and ghost in the free ground state will be considered in this subsection. Let us separately examine the two cases.

2.3.1 Gluons

Taking into account the expressions (A.4), the gluon annihilation and creation parts in (10) can be explicitly written as

$$A_{\mu}^{a+}(x) = \sum_{\mathbf{k}} \left(\sum_{\lambda=1,2} A_{\mathbf{k},\lambda}^a f_{k,\mu}^{\lambda}(x) + A_{\mathbf{k}}^{L,a} f_{k,L,\mu}(x) + B_{\mathbf{k}}^a \left[f_{k,S,\mu}(x) + (1-\alpha) (A(k_0, x_0) f_{k,L,\mu}(x) + B_{\mu}(k, x)) \right] \right), \quad (22)$$

$$A_{\mu}^{a-}(x) = \sum_{\mathbf{k}} \left(\sum_{\lambda=1,2} A_{\mathbf{k},\lambda}^{a+} f_{k,\mu}^{\lambda*}(x) + A_{\mathbf{k}}^{L,a+} f_{k,L,\mu}^*(x) + B_{\mathbf{k}}^{a+} \left[f_{k,S,\mu}^*(x) + (1-\alpha) (A^*(k_0, x_0) f_{k,L,\mu}^*(x) + B_{\mu}^*(k, x)) \right] \right),$$

where $A(k_0, x_0)$ and $B_{\mu}(k, x)$ are defined as

$$A(k_0, x_0) \equiv \frac{1}{2|\mathbf{k}|^2} (ik_0 x_0 + 1/2),$$

$$B_{\mu}(k, x) \equiv \frac{ik_0 \delta_{\mu 0}}{2|\mathbf{k}|^2} g_k(x).$$

On arriving to (22), the term of $\partial_{\mu} D_{(x)}^{(1/2)} g_k(x)$ in (A.4) was evaluated by using the wave packets and polarization vectors of (A.5) with the result

$$\begin{aligned} \partial_{\mu} D_{(x)}^{(1/2)} g_k(x) &= \partial_{\mu} 1/2(\nabla^2)^{-1} (x_0 \partial_0 - 1/2) \\ &\quad \times \frac{1}{\sqrt{2V} k_0} \exp(-ikx) \\ &= A(k_0, x_0) f_{k,L,\mu}(x) \\ &\quad + B_{\mu}(k, x), \end{aligned} \quad (23)$$

which defined $A(k_0, x_0)$ and $B_{\mu}(k, x)$. In a similar way, for $\partial_{\mu} D_{(x)}^{(1/2)} g_k^*(x)$ we obtained

$$\begin{aligned} \partial_{\mu} D_{(x)}^{(1/2)} g_k^*(x) &= A^*(k_0, x_0) f_{k,L,\mu}^*(x) \\ &\quad + B_{\mu}^*(k, x). \end{aligned} \quad (24)$$

As commented on in Appendix A, the transverse term is not modified by the generalization of the theory for arbitrary values of α . Then its contribution is the same as calculated previously [3, 5]. It is important to recall that thanks to the diagonal block structure commutation relations (A.6) the calculations are done by decomposing the expression (10) in products of contributions coming from separate modes associated to the diagonal blocks.

The result obtained in [3, 5] for the transverse-mode contributions, for $|C_{\lambda}(p_i)| < 1$, is

$$\begin{aligned} &\frac{1}{N} \langle 0 | \exp \left(\sum_{\lambda=1,2} \frac{C_{\lambda}^*(P)}{2} A_{\mathbf{p}_i,\lambda}^a A_{\mathbf{p}_i,\lambda}^a \right) \\ &\quad \times \exp \left\{ i \int dx J^{\mu,a}(x) \sum_{\lambda=1,2} A_{\mathbf{p}_i,\lambda}^{a+} f_{p_i,\mu}^{\lambda*}(x) \right\} \\ &\quad \times \exp \left\{ i \int dx J^{\mu,a}(x) \sum_{\lambda=1,2} A_{\mathbf{p}_i,\lambda}^a f_{p_i,\mu}^{\lambda}(x) \right\} \\ &\quad \times \exp \left(\sum_{\lambda=1,2} \frac{C_{\lambda}(P)}{2} A_{\mathbf{p}_i,\lambda}^{a+} A_{\mathbf{p}_i,\lambda}^{a+} \right) | 0 \rangle, \\ &= \exp \left\{ - \sum_{\lambda=1,2} (J_{p_i,\lambda}^a)^2 \frac{(C_{\lambda}(P) + C_{\lambda}^*(P) + 2|C_{\lambda}(P)|^2)}{2(1-|C_{\lambda}(P)|^2)} \right\}, \end{aligned} \quad (25)$$

where the normalization factor has been canceled. It should be recalled that the momenta \mathbf{p}_i are very small and the sources are assumed to be located in a finite spatial region. The following simplified notation was also introduced in [3, 5]:

$$j_{p_i,\lambda}^a = \int \frac{dx}{\sqrt{2V} p_{i0}} j^{\mu,a}(x) \varepsilon_{\lambda,\mu}(p_i).$$

The calculation of longitudinal and scalar-mode contributions is, on this occasion, more elaborate, and a brief exposition of it can be found in Appendix A; the result obtained for $|C_3(\alpha, P)| < 1$ is

$$\begin{aligned} &\exp \left\{ - \int \frac{dx dy}{2V p_{i0}} J^{\mu,a}(x) J^{\nu,a}(y) \right. \\ &\quad \times \left[\frac{(C_3(\alpha, P) + C_3^*(\alpha, P) + 2|C_3(\alpha, P)|^2)}{(1-|C_3(\alpha, P)|^2)} \right. \\ &\quad \times \left(\varepsilon_{S,\mu}(p_i) \varepsilon_{L,\nu}(p_i) + \frac{(1-\alpha)}{4|\mathbf{p}_i|^2} [\varepsilon_{L,\mu}(p_i) + i2p_{i0} \delta_{\mu 0}] \right. \\ &\quad \left. \left. \left. \times \varepsilon_{L,\nu}(p_i) \right) \right. \right. \\ &\quad \left. \left. \left. + \frac{(D^*(\alpha, P)(C_3(\alpha, P) + 1)^2 + D(\alpha, P)(C_3^*(\alpha, P) + 1)^2)}{(1-|C_3(\alpha, P)|^2)^2} \right) \right. \right. \\ &\quad \left. \left. \left. \times \varepsilon_{L,\mu}(p_i) \varepsilon_{L,\nu}(p_i) \right] \right\} \\ &\quad \times \langle 0 | \exp (C_3^*(\alpha, P) B_{\mathbf{p}_i}^a A_{\mathbf{p}_i}^{L,a} + D^*(\alpha, P) B_{\mathbf{p}_i}^a B_{\mathbf{p}_i}^a) \\ &\quad \times \exp (C_3(\alpha, P) B_{\mathbf{p}_i}^{a+} A_{\mathbf{p}_i}^{L,a+} + D(\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+}) | 0 \rangle, \end{aligned} \quad (26)$$

where the normalization factor (last line) is equal to one. Now inserting expressions (25) and (26) in (10), performing some algebraic manipulations (keeping in mind the

properties of the polarization vectors defined in (A.5)), assuming $C_1(P) = C_2(P)$, and introducing the contribution of all momenta \mathbf{p}_i ($|\mathbf{p}_i| = P$), the following expression is obtained:

$$\begin{aligned} & \exp \left\{ \int \frac{dxdy}{2V} J^{\mu,a}(x) J^{\nu,a}(y) \right. \\ & \times \sum_{\mathbf{p}_i, |\mathbf{p}_i|=P} \frac{1}{p_{i0}} \left[\left(\frac{C_1(P) + C_1^*(P) + 2|C_1(P)|^2}{2(1 - |C_1(P)|^2)} \right) g_{\mu\nu} \right. \\ & + \left(\frac{C_3(\alpha, P) + C_3^*(\alpha, P) + 2|C_3(\alpha, P)|^2}{(1 - |C_3(\alpha, P)|^2)} \right. \\ & \left. \left. - \frac{C_1(P) + C_1^*(P) + 2|C_1(P)|^2}{(1 - |C_1(P)|^2)} \right) \frac{\bar{p}_{i\mu} p_{i\nu}}{2|\mathbf{p}_i|^2} \right. \\ & + \left(\frac{C_3(\alpha, P) + C_3^*(\alpha, P) + 2|C_3(\alpha, P)|^2}{(1 - |C_3(\alpha, P)|^2)} \right) \frac{(1-\alpha)}{4|\mathbf{p}_i|^2} \\ & \times (p_{i\mu} p_{i\nu} - 2p_{i0} \delta_{\mu 0} p_{i\nu}) \\ & + \frac{D^*(\alpha, P) (C_3(\alpha, P) + 1)^2 + D(\alpha, P) (C_3^*(\alpha, P) + 1)^2}{(1 - |C_3(\alpha, P)|^2)^2} \\ & \left. \left. \times p_{i\mu} p_{i\nu} \right] \right\}. \end{aligned} \quad (27)$$

It is important to notice at this point that the combinations of $C_1(P)$, $C_3(\alpha, P)$, and $D^*(\alpha, P)$ in (27) are real. So that even if these parameters are complex, only their real parts contribute to the propagator. After this observation, they will be considered real in what follows.

In expression (27) we take the thermodynamic limit ($V \rightarrow \infty$), replace sums by integrals,

$$\frac{1}{V} \sum_{\mathbf{p}_i, |\mathbf{p}_i|=P} = \frac{1}{(2\pi)^3} \int_0^\infty dp p^2 \delta(p - P) \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi, \quad (28)$$

and perform the integrations.

As a second step, we proceed to take the limit $P \rightarrow 0$; for that we consider that the parameters introduced in the modified vacuum state to be

$$\begin{aligned} C_1(P) &\sim 1 - \frac{C_1}{2}P, \quad C_1 > 0, \\ C_3(\alpha, P) &\sim 1 - \frac{C_3(\alpha)}{2}P, \quad C_3(\alpha) > 0, \\ D(\alpha, P) &\sim D(\alpha). \end{aligned} \quad (29)$$

After that, the result obtained for (27) in the limit $P \rightarrow 0$ is

$$\begin{aligned} & \exp \left\{ \int dxdy J^{\mu,a}(x) J^{\nu,a}(y) \left[\frac{2g_{\mu\nu}}{(2\pi)^2 C_1} \right. \right. \\ & + \frac{2}{3(2\pi)^2} \left(\frac{1}{C_3(\alpha)} - \frac{1}{C_1} \right) (g_{\mu\nu} + 2\delta_{\mu 0} \delta_{\nu 0}) \\ & - \frac{(1-\alpha)}{(2\pi)^2 3C_3(\alpha)} (g_{\mu\nu} + 2\delta_{\mu 0} \delta_{\nu 0}) \\ & \left. \left. - \frac{4D(\alpha)}{(2\pi)^2 3[C_3(\alpha)]^2} (g_{\mu\nu} - 4\delta_{\mu 0} \delta_{\nu 0}) \right] \right\}. \end{aligned} \quad (30)$$

In order to make explicit the Lorentz invariance, we fix $D(\alpha)$, such that the terms proportional to $\delta_{\mu 0} \delta_{\nu 0}$ cancel out. Then $D(\alpha)$ is determined by the expression

$$D(\alpha) = \frac{[C_3(\alpha)]^2}{4} \left[\frac{1}{C_1} - \frac{(1+\alpha)}{2C_3(\alpha)} \right],$$

and (30) takes the form

$$\exp \left\{ \int dxdy J^{\mu,a}(x) J^{\nu,a}(y) \frac{g_{\mu\nu}}{(2\pi)^2} \left(\frac{1}{C_1} + \frac{(1+\alpha)}{2C_3(\alpha)} \right) \right\}. \quad (31)$$

In expression (31) we notice that the term in the exponential is real and non-negative. This is because $C_1 > 0$, $C_3(\alpha) > 0$ and $\alpha \geq 0$ (α must be non-negative for the convergence of the Gaussian integral in which it was introduced in the path integral formalism. It can be stressed at this point, that, if it could be possible to justify the validity of the recursive solution as an analytical extension in the parameters, it seems possible to trace a connection with the approach of [17, 18, 21]). As a final step, we chose the parameter $C_3(\alpha)$ in such a way that the determined modification does not depend on the gauge parameter α , as it was selected in an earlier work [2]. Then $C_3(\alpha) = \frac{1}{2}K C_1(1 + \alpha)$, with K a positive constant.

Considering the above remarks and defining in the generating functional modification an alternative non-negative constant $C_g = \frac{2(2\pi)^2}{C_1} (1 + \frac{1}{K})$ (which we call the condensate parameter) for further convenience, expression (31) takes the form

$$\exp \left\{ \int dxdy \sum_{a=1}^8 J^{\mu,a}(x) J^{\nu,a}(y) \frac{g_{\mu\nu} C_g}{(2\pi)^4 2} \right\}.$$

From the above construction, the parameter $D(\alpha) = D(1 + \alpha)^2$, where D is an arbitrary real constant (equal to zero for $K = 1$), and the vacuum in the zero-momentum limit has the form

$$\begin{aligned} |\Psi\rangle &= \exp \left[\sum_{a=1}^8 \frac{1}{2} A_{0,1}^{a+} A_{0,1}^{a+} + \frac{1}{2} A_{0,2}^{a+} A_{0,2}^{a+} + B_0^{a+} A_0^{L,a+} \right. \\ & \left. + i\bar{c}_0^{a+} c_0^{a+} + D(1 + \alpha)^2 B_0^{a+} B_0^{a+} \right] |0\rangle, \end{aligned} \quad (32)$$

from which the state selected in the previous works [3, 5] corresponds to the particular case $\alpha = 1$, $C_3(\alpha = 1) = C_1$ and $D(\alpha = 1) = 0$.

Thus, it follows that the vacuum state introduced in (32) modifies the usual perturbation theory only through a change in the gluon propagator, which now takes the form

$$D_{\mu\nu}^{ab}(x-y) = \int \frac{dk}{(2\pi)^4} \delta^{ab} \left[\frac{\theta(|\mathbf{k}| - \sigma)}{k^2 + i\varepsilon} \times \left(g_{\mu\nu} - (1-\alpha) \frac{k_\mu k_\nu}{k^2 + i\varepsilon} \right) - iC_g \delta(k) g_{\mu\nu} \right] \times \exp\{-ik(x-y)\}. \quad (33)$$

in which the first term is the usual Feynman propagator, but now including an infrared regularization.

2.3.2 Ghosts

The change in the ghost generating functional is the same as calculated in [3, 5]. This is because the ghost sector in the vacuum state remains unchanged. As was mentioned in these references for the value $C_3(\alpha, 0) = 1$ as it was fixed here, there is no modification of the ghost sector and its generating functional and propagator remain the same as for the usual PQCD.

2.3.3 Quarks

Let us now consider the modification in the quark propagator produced by the presence of its condensate. The factor corresponding to quarks in the generating functional (3) can be written as

$$\begin{aligned} Z_0^{q,m}[\eta, \bar{\eta}] &= \langle 0 | \exp \left(\sum_{s=1,2} \sum_{p_i, q_i, |p_i, q_i| = P < \sigma} C_{\mathbf{p}_i, \mathbf{q}_i}^* a_{\mathbf{p}_i}^s b_{\mathbf{q}_i}^s \right) \\ &\times \exp \left\{ i \int dx \left[\bar{\eta}(x) \psi^-(x) + \bar{\psi}^-(x) \eta(x) \right] \right\} \\ &\times \exp \left\{ i \int dx \left[\bar{\eta}(x) \psi^+(x) + \bar{\psi}^+(x) \eta(x) \right] \right\} \\ &\times \exp \left(\sum_{s=1,2} \sum_{p_i, q_i, |p_i, q_i| = P < \sigma} C_{\mathbf{p}_i, \mathbf{q}_i} b_{\mathbf{q}_i}^{s+} a_{\mathbf{p}_i}^{s+} \right) |0\rangle \\ &= \langle 0 | \exp \left[C_{ff'}^* a_f^s b_{f'}^s \right] \exp \left\{ -\sigma_f a_f^+ + \bar{\sigma}_f b_f^+ \right\} \\ &\times \exp \left\{ -\sigma_f^* a_f + \bar{\sigma}_f^* b_f \right\} \exp \left[C_{ff'} b_{f'}^{s+} a_f^{s+} \right] |0\rangle \\ &= Z_0^{q,m}[\sigma, \bar{\sigma}] \equiv Z_0^{q,m}[v], \\ v &\equiv (\sigma, \bar{\sigma}), \quad |q\rangle = \exp \left[C_{ff'} b_{f'}^{s+} a_f^{s+} \right] |0\rangle, \quad (34) \end{aligned}$$

where the quark-field operators were expressed as sums over the creation and annihilation components. In order to simplify the discussion the following compact notation has been employed: $f, f' = \mathbf{p}_i, i = 1, 2, 3, \dots, N$. The new indices, f and f' , correspond to the values of the momenta

for which condensate states are created. The spinor indices $s = 1$ or 2 have been omitted, since the discussion can be done almost up to the end for each value of s , thanks to the commutativity of the creation operators for different indices. The coefficients of the quark creation and annihilation operators a^\pm and b^\pm are expressed as

$$\begin{aligned} \sigma_f &\equiv \sigma_{\mathbf{k}}^s = \frac{i}{\sqrt{V}} \int dx \bar{u}_{\mathbf{k}}^s \eta(x) \exp(ikx), \\ \bar{\sigma}_f &\equiv \bar{\sigma}_{\mathbf{k}}^s = \frac{i}{\sqrt{V}} \int dx \bar{\eta}(x) \bar{v}_{\mathbf{k}}^s \exp(ikx), \quad (35) \end{aligned}$$

where u and v are the usual spinor solutions of the Dirac equation defined in Appendix A.

Let us now define the transformations $U_1^{-1} = \exp(-\sigma_f a_f^+ + \bar{\sigma}_f b_f^+)$ and $U_2 = \exp(-\sigma_f^* a_f + \bar{\sigma}_f^* b_f)$, which, acting upon the creation and annihilation operators through a similarity, lead to

$$\begin{aligned} U_1 b_f U_1^{-1} &= b_f - \bar{\sigma}_f, & U_1 a_f U_1^{-1} &= a_f + \sigma_f, \\ U_2 b_f^+ U_2^{-1} &= b_f^+ + \bar{\sigma}_f^*, & U_2 a_f^+ U_2^{-1} &= a_f^+ - \sigma_f^*. \end{aligned}$$

After also considering $\langle 0 | U_1 = \langle 0 |$ and $U_2 | 0 \rangle = |0\rangle$, it follows that

$$\begin{aligned} Z_0^{q,m}[v] &= \langle 0 | \exp \left[C_{ff'}^* (a_f + \sigma_f) (b_{f'} - \bar{\sigma}_{f'}) \right] \\ &\times \exp \left[C_{ff'} (b_{f'}^{s+} + \bar{\sigma}_{f'}^*) (a_f^{s+} - \sigma_f^*) \right] |0\rangle, \quad (36) \\ &= \langle q | \exp \left[-C_{ff'}^* \sigma_f \bar{\sigma}_{f'} - C_{ff'} \bar{\sigma}_{f'}^* \sigma_f^* \right] \\ &\times \exp \left[-a_f C_{ff'} \bar{\sigma}_{f'} + \sigma_f C_{ff'} b_{f'} \right] \\ &\times \exp \left[-a_f^* C_{ff'} \bar{\sigma}_{f'}^* + \sigma_f^* C_{ff'} b_{f'}^+ \right] |q\rangle. \end{aligned}$$

It is now possible to employ the following identity for operators that commute with their commutators:

$$\exp(A) \exp(B) = \exp(B) \exp(A) \exp([A, B]), \quad (37)$$

to change the order of the exponential operators appearing in the mean value on the quark ‘‘squeezed’’ state in (36). The commutator between the two arguments of those exponentials can be calculated to be

$$\begin{aligned} &\left[-a_f C_{ff'} \bar{\sigma}_{f'} + \sigma_f C_{ff'} b_{f'}, -a_f^* C_{ff'} \bar{\sigma}_{f'}^* + \sigma_f^* C_{ff'} b_{f'}^+ \right] \\ &= - \left[\bar{\sigma}_{f'} C_{ff'}^T C_{ff''} \bar{\sigma}_{f''} + \sigma_f C_{ff'}^* C_{ff''}^T \sigma_{f''}^* \right], \end{aligned}$$

which, after being considered together with (37), allows us to write the quark modification as

$$\begin{aligned} Z_0^{q,m}[v] &= \exp \left[-\sigma \hat{C} \bar{\sigma} - \bar{\sigma}^* \hat{C} \sigma^* - \bar{\sigma} \hat{C}^T \hat{C} \bar{\sigma}^* - \sigma \hat{C}^T \hat{C} \sigma^* \right] \\ &\times Z \left[-\hat{C} \bar{\sigma}^*, \hat{C}^T \sigma^* \right], \quad (38) \\ &= \exp \left[\frac{1}{2} v_1 R v_1 - \frac{1}{2} v_1^* R v_1^* + v_1 R^2 v_1^* \right] Z_q [R \mathcal{K}_{v_1}], \end{aligned}$$

where it was assumed that $C_{ff'}$ is real, and the following matrix notation was introduced:

$$\begin{aligned}\sigma\widehat{C}\bar{\sigma} &\equiv \sigma_f C_{ff'} \bar{\sigma}_{f'}, & (\widehat{C}^T)_{ff'} &= (\widehat{C})_{f'f}, \\ \mathbf{v}_1 &= \begin{pmatrix} \sigma \\ \bar{\sigma} \end{pmatrix}, & R &= \begin{pmatrix} 0 & -\widehat{C} \\ \widehat{C} & 0 \end{pmatrix}, \\ \mathcal{K}\mathbf{v}_1 &= \begin{pmatrix} \sigma^* \\ \bar{\sigma}^* \end{pmatrix}, & \mathcal{K}^2 &= I,\end{aligned}$$

and \mathcal{K} is simply the complex conjugate of a vector expressed in matrix notation. Relation (38), after being applied n times, leads to the following expression:

$$\begin{aligned}Z_0^{q,m}[\mathbf{v}] &= \exp\left\{\frac{1}{2}\mathbf{v}_1 R \left(\sum_{m=0}^n R^{2m}\right) \mathbf{v}_1 - \frac{1}{2}\mathbf{v}_1^* R\right. \\ &\quad \times \left.\left(\sum_{m=0}^n R^{2m}\right) \mathbf{v}_1^* + \mathbf{v}_1 R^2 \left(\sum_{m=0}^n R^{2m}\right) \mathbf{v}_1^*\right\} \\ &\quad \times Z_q[(\mathcal{K}R)^{n+1} \mathbf{v}_1].\end{aligned}$$

However, the sums of powers of the matrix R can be written in the form

$$\sum_{m=0}^n R^{2m} = \begin{pmatrix} \sum_{m=0}^n (-1)^m (\widehat{C}\widehat{C}^T)^m & 0 \\ 0 & \sum_{m=0}^n (-1)^m (\widehat{C}^T\widehat{C})^m \end{pmatrix},$$

which gives in the limit $n \rightarrow \infty$, assuming that all the eigenvalues of the symmetric matrix R^2 are smaller than 1:

$$\sum_{m=0}^{\infty} R^{2m} = \begin{pmatrix} \frac{1}{1+\widehat{C}\widehat{C}^T} & 0 \\ 0 & \frac{1}{1+\widehat{C}^T\widehat{C}} \end{pmatrix} = \frac{1}{1-R^2}.$$

Hence, the modification of the quark propagator takes the form

$$\begin{aligned}Z_0^{q,m}[\mathbf{v}] &= \exp\left\{\frac{1}{2}\mathbf{v}_1 \frac{R}{1-R^2} \mathbf{v}_1 + \frac{1}{2}\mathbf{v}_1^* \frac{R}{1-R^2} \mathbf{v}_1^*\right. \\ &\quad \left.+ \mathbf{v}_1 \frac{R^2}{1-R^2} \mathbf{v}_1^*\right\} Z_q[0],\end{aligned}\quad (39)$$

where the argument of the generating functional at r.h.s. is zero because $\lim_{n \rightarrow 0} (R^{n+1}) = 0$, if all the eigenvalues of R^2 are assumed to be smaller than the unity. For the argument of the exponential above it follows that

$$\mathbf{v}_1 \frac{R^2}{1-R^2} \mathbf{v}_1^* = -\sigma \frac{\widehat{C}\widehat{C}^T}{1+\widehat{C}\widehat{C}^T} \sigma^* - \bar{\sigma} \frac{\widehat{C}^T\widehat{C}}{1+\widehat{C}^T\widehat{C}} \bar{\sigma}^*.$$

Now, assuming that the matrix C is antisymmetric and also that it satisfies $\widehat{C}^T \widehat{C} = K\widehat{I}$ (with K including also possible negative values), it follows that the sum of the first

two terms in the argument of the exponential in (39) vanishes. The one remaining takes the form

$$\begin{aligned}v_1 \frac{R^2}{1-R^2} v_1^* &= \frac{K}{1-K} (\sigma\sigma^* + \bar{\sigma}\bar{\sigma}^*) = \frac{K}{1-K} (\sigma_f\sigma_f^* + \bar{\sigma}_f\bar{\sigma}_f^*) \\ &= \frac{K}{1-K} \sum_{\mathbf{k}} (\sigma_{\mathbf{k}}^s \sigma_{\mathbf{k}}^{s*} + \bar{\sigma}_{\mathbf{k}}^s \bar{\sigma}_{\mathbf{k}}^{s*}),\end{aligned}$$

where the index $s = 1, 2$ takes the fixed value that has been used for the whole evaluation. Now employing the definitions (35) of σ and $\bar{\sigma}$ and the identities

$$\sum_{s=1,2} v_{\mathbf{k}}^s \bar{v}_{\mathbf{k}}^s = (\gamma^\mu p_\mu - m), \quad \sum_{s=1,2} u_{\mathbf{k}}^s \bar{u}_{\mathbf{k}}^s = (\gamma^\mu p_\mu + m),$$

and assuming that the momenta arguments p_i are taken at their vanishing value in the limit $L \rightarrow \infty$, it follows that

$$\begin{aligned}\frac{K}{1-K} \sum_{\mathbf{k}, s=1,2} (\sigma_{\mathbf{k}}^s \sigma_{\mathbf{k}}^{s*} + \bar{\sigma}_{\mathbf{k}}^s \bar{\sigma}_{\mathbf{k}}^{s*}) \\ = -\frac{K}{1-K} \frac{m}{V} \sum_{\mathbf{k}} \int dx dy \bar{\eta}(x) \eta(y) \exp(ik(x-y)), \\ = -\frac{K}{1-K} \frac{m}{V} N \int dx dy \bar{\eta}(x) \eta(y) \exp(ik(x-y)),\end{aligned}$$

where the number N of the momenta for the created particles and antiparticles appears in the last line from the assumption that for small momenta the integral appearing is almost independent of k . Fixing K by

$$K = \frac{1}{1 - \frac{m(L)N(2\pi)^4}{C_q L^3}}$$

and considering (20) allows us to write for the change in the quark generating functional and propagators

$$Z_0^{q,m}[\eta, \bar{\eta}] = \exp\left(C_q \int dx dy \bar{\eta}(x) \eta(y)\right) Z_q[0], \quad (40)$$

$$Z_0^{q,F}[\eta, \bar{\eta}] = \exp\left(\int dx dy \bar{\eta}(x) G_F(x-y) \eta(y)\right) Z_q[0], \quad (41)$$

$$Z_0^q[\eta, \bar{\eta}] = \exp\left(\int dx dy \bar{\eta}(x) G^q(x-y) \eta(y)\right) Z_q[0], \quad (42)$$

$$\begin{aligned}G^q(x-y) &= \int \frac{dk}{(2\pi)^4} \left[-\frac{i\theta(|\mathbf{k}|-\sigma)\gamma^\nu k_\nu}{k^2 + i\varepsilon} + C_q I \delta(k) \right] \\ &\quad \times \exp(-ik(x-y)).\end{aligned}\quad (43)$$

3 Regularization of the singular terms

A main technical difficulty for the implementation of the expansion proposed in [2–4, 6, 16] is that the modifications of the usual free propagator terms (to be called below

the ‘‘condensate propagators’’) are given by Dirac’s delta functions of the momenta. This circumstance produces singular Feynman diagrams in the perturbation series even after dimensional regularization is introduced. The singularities correspond with the appearance of delta functions or standard Feynman propagators evaluated at zero momentum, after some loop integrals are performed. These factors occur because of the momentum conservation in the vertices of the diagrams. Let us consider a vertex having n legs ($n = 3, 4$ for QCD). Then, when $n - 1$ different condensate lines join to it, the momentum conservation forces the value of the momentum at the only remaining line to vanish. Therefore, if a condensate line is attached to this ending, delta functions evaluated at zero momentum will appear. On the other hand, when a usual Feynman propagator is connected to this last leg, a factor equal to its value at zero momentum appears. This situation should be solved before a sense could be given to the modified expansion. Below, we propose a way for the elimination of these singularities, which clearly should be the subject of further examination about its consistency. Let us separately consider in what follows the two cases. It will be assumed that the Feynman diagrams are constructed in D dimensions and for a fixed value of the Feynman parameter ε . Before advancing, let us precisely set the rules for the order of the limits associated to the various regularizations that have been done up to now.

- (a) The large volume limit will be taken by taking the size of the quantization box L going to infinity. This limit will lead to continuous values of the momenta.
- (b) The Nakanishi infrared regularization parameter σ will be chosen as a function $\sigma(L)$ of the spatial size of the system L , vanishing as $L \rightarrow \infty$. This constant sets to zero all the Feynman propagators for the spatial momenta lying within a sphere of radius σ , and all values of the temporal component k_0 .
- (c) The maximal size P of the set of the spatial momenta \mathbf{k}_i for which condensate states are created will be assumed to be another function $P(L)$ also vanishing in the limit $L \rightarrow \infty$. However, it will be chosen smaller than $\sigma(L)$, in a proportion to be defined in the following.
- (d) Finally, the auxiliary mass m of the quark field will be chosen as a function $m(L)$ tending to zero for $L \rightarrow \infty$ with a behavior to be also described below.

The limit $L \rightarrow \infty$, in which $\sigma(L)$, $P(L)$, and $m(L)$ vanish, will be taken in the first place. With this step we recover the Lorentz invariance of the Feynman diagrams that continue to be functions of the dimensional regularization parameter D , its mass scale μ , and the Feynman constant ε .

3.1 $\delta(0)$ singularities

An idea that comes directly to mind when considering the singular terms of the form $\delta(0)$ is that they should be considered in dimensional regularization. But it has been argued that the delta functions evaluated at zero spatial

coordinates can be analytically extended to continuous dimensions D and, moreover, that their expressions vanish in the limit $D \rightarrow 4$ [26]. This is not necessarily an un-natural result, as it is suggested by the fact that the product of regularized functions in the sequences defining distributions, can have a vanishing limiting result for particular constructions of those sequences, in spite of the singularity of the distributions being defined. Thus, let us examine more closely this dimensional regularization argument. We will follow the same procedure as in [26] and interpret the delta functions appearing as D -dimensional ones. The Wick rotation of the momentum integrals will be assumed to be already done. Then it is possible to reproduce, step by step, the arguments of Capper and Leibbrandt [26] to conclude that these factors should vanish after removing the dimensional regularization. Let us do it below for the sake of concreteness. For $\delta(0)$ we can write

$$\delta(0) = \int_E \frac{dp^D}{(2\pi)^D}.$$

This is a singular D -dimensional integral in Euclidean momentum space, similar to those in real space, considered in [26]. Then it can also be written as follows:

$$\int_E \frac{dp^D}{(2\pi)^D} = \int_E \frac{dp^D}{(2\pi)^D} \frac{p^2}{p^2} = \int_0^\infty ds \int_E \frac{dp^D}{(2\pi)^D} \frac{p^2}{p^2} \exp(-s p^2).$$

However, using the redefinition of the generalized Gaussian integral for continuous values of dimension D constructed in [26], it is possible to write

$$\begin{aligned} \int_E \frac{dp^D}{(2\pi)^D} \exp(-s p^2) &= \frac{1}{(4\pi)^{\frac{D}{2}}} \exp\left[-s f\left(\frac{D}{2}\right)\right], \\ \int_E \frac{dp^D}{(2\pi)^D} p^2 \exp(-s p^2) &= \frac{1}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2} s^{-(1+\frac{D}{2})} + s^{-\frac{D}{2}} f\left(\frac{D}{2}\right) \right] \exp\left[-s f\left(\frac{D}{2}\right)\right], \end{aligned}$$

and

$$\begin{aligned} \delta(0) &= \int_0^\infty ds \frac{1}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2} s^{-(1+\frac{D}{2})} + s^{-\frac{D}{2}} f\left(\frac{D}{2}\right) \right] \\ &\quad \times \exp\left[-s f\left(\frac{D}{2}\right)\right], \end{aligned}$$

where f is the function introduced in [26] for extending the generalized Gaussian integral formula to non-integral dimension arguments. These functions vanish for all integer values of D . Then, after using the integral definition of the Gamma function $\Gamma(z) = \int_0^\infty dt t^{z-1} \exp(-t)$, it follows that

$$\delta(0) = \frac{f(\frac{D}{2})^D}{(4\pi)^{\frac{D}{2}}} \left[\frac{D}{2} \Gamma\left(-\frac{D}{2}\right) + \Gamma\left(1 - \frac{D}{2}\right) \right],$$

which vanishes exactly in the limit $D \rightarrow 4$. Therefore, we will assume that the evaluations associated to the modified expansion will be made by using the above representation

for the factors $\delta(0)$. As a consequence, it will be considered that all the diagrams in which such a kind of singularities appear will vanish in dimensional regularization.

3.2 Feynman propagator at $p = 0$ singularities

For the elimination of this kind of singular behavior, the Nakanishi infrared regularization implemented on the Feynman propagators in a previous section is specially helpful. These singular terms are simply vanishing because of the appearance of the infrared regularizing factor $\theta(|\vec{p}| - \sigma)$ in each of the Feynman propagators, before taking the limit $L \rightarrow \infty$.

Let us examine below the loop expansion after assuming the above prescriptions and the modified free propagators for a gauge parameter α

$$\begin{aligned} D_{g\mu\nu}^{ab}(p, m) &= \frac{\theta(|\mathbf{p}| - \sigma)\delta^{ab}}{p^2 + i\varepsilon} \left[g_{\mu\nu} - \frac{(1-\alpha)p_\mu p_\nu}{p^2 + i\varepsilon} \right] \\ &\quad - iC_g \delta^{ab} \delta(p), \\ G_q^{f_1 f_2}(p, M, S) &= -\frac{\theta(|\mathbf{p}| - \sigma)\delta^{f_1 f_2} p_\mu \gamma^\mu}{p^2 + i\varepsilon} + i\delta^{f_1 f_2} C_q \delta(p), \\ G_{gh}^{ab}(p) &= -\frac{\theta(|\mathbf{p}| - \sigma)\delta^{ab}}{p^2 + i\varepsilon}, \end{aligned} \quad (44)$$

and the standard vertices of QCD. Then, it follows that all the diagrams having a fixed number of loops showing that both types of singularities will vanish in the limit $L \rightarrow \infty$. Therefore, after taking this limit, the remaining finite diagrams (due to the dimensional regularization) can be evaluated by using non-distorted dimensionally regularized propagators, following from (44) in the limit $\sigma \rightarrow 0$.

Assuming that the chosen conditions have eliminated all the singular contributions, it is possible to comment about some properties of the diagram series based on the above propagators. For example, it follows that the appearance of a number m of the condensate propagators within an n -loop one-particle-irreducible (1PI) diagram will eliminate m of the n -loop integrals associated to this contribution. Therefore, the considered diagram will now be an ‘‘effective’’ $(n - m)$ -loop one. Consider also, expressing the condensate parameters C_g and C_q in favor of the new ones:

$$m^2 = -\frac{6g^2 C_g}{(2\pi)^4}, \quad S_f = \frac{g^2 C_q}{4\pi^4}, \quad (45)$$

which incorporate a power of 2 factor of the coupling constant g . Then, the n -loop 1PI diagrams of the effective expansion, considered as power series in the new three parameters m^2 , S_f and g , also show the property that, given the number of external legs of the diagram, the number of loops is fixed by the power of g^2 appearing in it. This conclusion directly follows from the fact that each time a condensate internal line appears, a loop integral is annihilated, and correspondingly the power of g of the diagram is reduced by 2 in the new expansion. Let us consider that P_g is the power of g corresponding to an n -loop 1PI diagram in which the parameters are the original ones. Thus

the new power of g of this diagram with m condensate lines when the new parameters are introduced will be

$$P'_g = P_g - 2m. \quad (46)$$

It seems that this property can allow for a useful reordering of the perturbation expansion. To see an indication for this, let us consider a particular n -loop 1PI diagram in which the changes of the parameters (45) have been introduced and corresponding line symbols have been assigned for the standard and the condensate propagators separately. Then, for any particular standard type line in this diagram, consider the infinite summation of all the contributions to the connected propagator of zero order in g (tree). Any of the added terms, by construction, have the same number of loops, but come from higher loops in the original expansion. This is done by considering fixed the other standard lines. Thus, if not stopped by any difficulty associated to the combinatorial and symmetry factors in the diagrams, this infinite addition, it seems, can be done for all the normal lines. The resulting modified (zero order in the new expansion) propagators are no other things than the ones employed in [6] in the gauge $\alpha = 1$. Therefore, it seems possible to demonstrate that the loop expansion can be reordered to produce an alternative version with modified propagators. The investigation of this possibility will be considered in our next work.

4 WTS identities

This section is devoted to showing that the generating functional constructed with the modified propagators, including condensate terms and the gauge parameter α , satisfies exactly the same WTS identities as those associated to the usual PQCD. Let us consider for the cited purpose the complete generating functional of the theory, which can be written in the form

$$\begin{aligned} Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \exp \left\{ i \left(\frac{S_{ikl}^g}{3!i^3} \frac{\delta^3}{\delta j_i \delta j_k \delta j_l} + \frac{S_{iklm}^g}{4!i^4} \frac{\delta^4}{\delta j_i \delta j_k \delta j_l \delta j_m} \right. \right. \\ &\quad \left. \left. + \frac{S_{ikl}^q}{i^3} \frac{\delta^3}{\delta \bar{\eta}_i \delta j_k \delta (-\eta_l)} + \frac{S_{ikl}^{gh}}{i^3} \frac{\delta}{\delta \bar{\xi}_i \delta j_k \delta (-\xi_l)} \right) \right\} \\ &\quad \times Z_0[j, \eta, \bar{\eta}, \xi, \bar{\xi}], \end{aligned} \quad (47)$$

in which the concrete form of the action S assumed and its conventions are defined in the Appendix A, and DeWitt compact notation for the integration and summation over the corresponding spatial, color or spinor indices was employed. The specific set of indices associated to the Latin letters i, k, l is specified in a natural way by the particular type of field to which it is associated as a subindex. For example in j_i the meaning is $i \equiv (x, \mu, a)$. The free generating functional (3) can also be expressed as a mean value in the initial state $|\Psi\rangle$ of the evolution operator U of the

interaction Lagrangian of the external sources, as follows:

$$\begin{aligned}
Z_0 [j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \lim_{T \rightarrow \infty} \sum_{n, n'} \langle \Psi | U(+T, -T) | j, \eta, \bar{\eta}, \xi, \bar{\xi} \rangle | \Psi \rangle, \\
&= \langle \Psi | T \exp \left\{ i \int dx \left[j \hat{A} + \bar{\xi} \hat{c} + \bar{c} \xi + \bar{\eta} \hat{\psi} \right. \right. \\
&\quad \left. \left. + \hat{\psi} \eta \right] \right\} | \Psi \rangle, \\
&= Z_0 [0, 0, 0, 0] \exp \left\{ i \frac{j_i D_g^{ik} j_k}{2} + i \bar{\eta}_i G_{gh}^{ik} \eta_k \right. \\
&\quad \left. + i \bar{\xi}_i G_q^{ik} \xi_k \right\}.
\end{aligned}$$

Consider now two complete bases of eigenstates $|q^\pm\rangle$ of the field operators taken at the fixed times T and $-T$. Thus, for these states

$$\begin{aligned}
\hat{A}^\alpha_{\mu}(\pm T, \mathbf{x}) |q^\pm\rangle &\equiv A_{q^\pm, \mu}^\alpha(\mathbf{x}) |q^\pm\rangle, \\
\hat{c}^\alpha(\pm T, \mathbf{x}) |q^\pm\rangle &\equiv c_{q^\pm}^\alpha(\mathbf{x}) |q^\pm\rangle, \\
\hat{\bar{c}}^\alpha(\pm T, \mathbf{x}) |q^\pm\rangle &\equiv \bar{c}_{q^\pm}^\alpha(\mathbf{x}) |q^\pm\rangle, \\
\hat{\psi}^\alpha(\pm T, \mathbf{x}) |q^\pm\rangle &\equiv \psi_{q^\pm}^\alpha(\mathbf{x}) |q^\pm\rangle, \\
\hat{\bar{\psi}}^\alpha(\pm T, \mathbf{x}) |q^\pm\rangle &\equiv \bar{\psi}_{q^\pm}^\alpha(\mathbf{x}) |q^\pm\rangle.
\end{aligned}$$

The explicit construction of the field operators for gauge theory in the holomorphic representation is done in [29]. This representation is the most convenient one for the construction of the basis states $|q^\pm\rangle$ [30]. Inserting the completeness relation at the instants $-T$ and T leads to

$$\begin{aligned}
Z_0 [j, \eta, \bar{\eta}, \xi, \bar{\xi}] &= \lim_{T \rightarrow \infty} \sum_{q, q^+} \langle \Psi | q^+ \rangle \langle q^+ | U(+T, -T) | j, \eta, \bar{\eta}, \xi, \bar{\xi} \rangle | q^- \rangle \langle q^- | \Psi \rangle \\
&= \lim_{T \rightarrow \infty} \sum_{q, q^+} \langle \Psi | q^+ \rangle \frac{1}{\mathcal{N}} \int \mathcal{D}\Phi \exp \left\{ i \int_{-T}^T dt \int dx \right. \\
&\quad \left. \times \left(\frac{1}{2} A_\mu^\alpha \left[\partial^2 g^{\mu\nu} - \left(1 - \frac{1}{\alpha} \right) \partial^\mu \partial^\nu \right] A_\mu^\alpha \right) \right\} \quad (48) \\
&\quad \times \exp \left\{ i \int_{-T}^T dt \int dx \left(-i \partial_\nu \bar{c}^\alpha \partial^\nu c + \bar{\psi} (i \gamma^\nu \partial_\nu - m) \psi \right. \right. \\
&\quad \left. \left. + j A + \bar{\xi} c + \bar{c} \xi + \bar{\eta} \psi + \bar{\psi} \eta \right) \right\} \langle q^- | \Psi \rangle,
\end{aligned}$$

where $\mathcal{D}\Phi$ means the path integral differential $\mathcal{D}\{A, c, \bar{c}, \psi, \bar{\psi}\}$, and the usual representation for the matrix element $\langle q^+ | U(+T, -T) | j, \eta, \bar{\eta}, \xi, \bar{\xi} \rangle | q^- \rangle$ of the evolution operator between eigenstates of the fields has been used [29]; \mathcal{N} is the usual source-independent normalization constant fixing $Z_0[0] = 1$. The $(q^-$ and $q^+)$ -dependence of the functional integral is contained in the boundary conditions of the field integration variables, which should be

fixed to the eigenvalues $A_{q^\pm, \mu}^\alpha, c_{q^\pm}^\alpha, \bar{c}_{q^\pm}^\alpha, \psi_{q^\pm}^\alpha, \bar{\psi}_{q^\pm}^\alpha$ at the times $\pm T$, respectively.

By inserting (48) in (47), and acting with the interaction part of the action (expressed in terms of the functional derivatives) on Z_0 , it follows that

$$\begin{aligned}
Z &= \exp \left\{ i \left[\frac{S_{ikl}^g}{3! i^3} \frac{\delta^3}{\delta j_i \delta j_k \delta j_l} + \frac{S_{iklm}^g}{4! i^4} \frac{\delta^4}{\delta j_i \delta j_k \delta j_l \delta j_m} \right. \right. \\
&\quad \left. \left. + \frac{S_{ikl}^{gh}}{2! i^3} \frac{\delta^3}{\delta \bar{\eta}_i \delta j_k \delta (-\eta_l)} + \frac{S_{ikl}^q}{i^3} \frac{\delta}{\delta \bar{\xi}_i \delta j_k \delta (-\xi_l)} \right] \right\} \\
&\quad \times Z_0 [j, \eta, \bar{\eta}, \xi, \bar{\xi}] \\
&= \lim_{T \rightarrow \infty} \sum_{q, q^+} \langle \Psi | q^+ \rangle \int \mathcal{D}\Phi \\
&\quad \times \exp \left\{ i \int_{-T}^T \int dt dx \left(\frac{1}{4} F_{\mu\nu}^\alpha F^{\mu\nu, \alpha} - \frac{1}{2\alpha} \partial^\nu A_\nu^\alpha \partial^\mu A_\mu^\alpha \right. \right. \\
&\quad \left. \left. - i \partial_\nu \bar{c}^\alpha D^{ab, \nu} c^b \right) \right\} \quad (49) \\
&\quad \times \exp \left\{ i \int_{-T}^T \int dt dx \left(\bar{\psi} (i \gamma^\nu D_\nu - m) \psi + j_\mu A^\mu + \bar{\xi} c \right. \right. \\
&\quad \left. \left. + \bar{c} \xi + \bar{\eta} \psi + \bar{\psi} \eta \right) \right\} \langle q^- | \Psi \rangle.
\end{aligned}$$

It can be noted that performing the derivatives over the sources of the interaction Lagrangian inside the integrand of the Gaussian functional integral is equivalent to adopting a perturbative definition of the functional integral for the interacting theory, as discussed in [31].

That is, the classical action defining the complete generating functional as a functional integral is exactly the same as for the usual PQCD. The changes in the diagrammatic expansion are thus only associated to the boundary conditions at $-T$ and $+T$. It should be noted that for the case of PQCD the wave functions $\Phi(q^\pm) = \langle q^\pm | 0 \rangle$ are specific Gaussian functions of the field components. It can be shown that the $\langle q^\pm | \Psi \rangle$ are also Gaussian wave functions, but defined by a different quadratic form as the argument of the exponentials.

Therefore, it can be concluded that all Ward identities of the usual PQCD should also be satisfied in the modified expansion. Since the action is identical, it therefore has the same BRST invariance transformation:

$$\begin{aligned}
\delta A_\mu^\alpha(x) &= \delta \lambda D_\mu^{ab} c^b(x), \\
\delta c^\alpha(x) &= -\frac{g}{2} \delta \lambda f^{abc} c^b(x) c^c(x), \quad \delta \bar{c}^\alpha(x) = -i \delta \lambda \partial_\mu A^{\mu, \alpha}(x), \\
\delta \psi(x) &= i g \delta \lambda c^\alpha(x) T_a \psi(x), \quad \delta \bar{\psi}(x) = -i g \delta \lambda c^\alpha(x) \bar{\psi}(x) T_a,
\end{aligned}$$

which, thanks to its Jacobian, equal to the unit, and after having been applied as a change of variables in (49), leads

to the functional form of the WTS identities:

$$\int dx \left\{ j^\mu(x) D_\mu^{ab} \frac{\delta}{i\delta j(x)} \frac{\delta}{i\delta \xi^b(x)} \right. \\ + \frac{g}{2} \bar{\xi}^a(x) f^{abc} \frac{\delta}{i\delta \bar{\xi}^a(x)} \frac{\delta}{i\delta \xi^a(x)} \\ - i\xi^a(x) \partial_\mu \frac{\delta}{i\delta j_\mu^a(x)} - ig \frac{\delta}{i\delta \eta^r(x)} \frac{\delta}{i\delta \bar{\xi}^a(x)} T_a^{rs} \eta^s(x) \\ \left. - ig \bar{\eta}^s(x) \frac{\delta}{i\delta \bar{\xi}^a(x)} T_a^{sr} \frac{\delta}{i\delta \eta^r(x)} \right\} Z[j, \eta, \bar{\eta}, \xi, \bar{\xi}] = 0.$$

To satisfy these relations implies that the mean values for physical quantities should be gauge invariant, as was demonstrated in general form in [23–25].

It should be remarked that the proof of the gauge invariance of the S matrix heavily relies on the existence of a mass shell defining the asymptotic states for the particles. But precisely QCD is expected to show the absence of these exact mass shells for its elementary fields: the quarks and gluons. As has already been evaluated in [4], these mass shells for quark and gluons are not following in the first approximation in the condensate parameters and coupling constant.

In ending this section, it can be noticed that the above outcome was suggested by the fact that the new propagators also are inverse kernels of the equations of motion of the free fields in massless QCD, as indicated by the relations

$$\left[k^2 g_{\mu\alpha} - \left(1 - \frac{1}{\alpha}\right) k_\mu k_\alpha \right] \\ \times \left\{ \frac{1}{k^2 + i\varepsilon} \left[g_\nu^\alpha - \frac{(1-\alpha)k^\alpha k_\nu}{(k^2 + i\varepsilon)} \right] - ig_\nu^\alpha C_g \delta(k) \right\} = g_{\mu\nu}, \\ -\gamma_\mu k^\mu \left[\frac{-\gamma_\mu k^\mu}{k^2 + i\varepsilon} + iC_f I \delta(k) \right] = I, \quad k^2 \left(\frac{1}{k^2 + i\varepsilon} \right) = I. \quad (51)$$

5 Gauge invariance in order g^2

Let us now study the validity of the general gauge-invariance properties that were obtained by means of the formal path integral procedures. The new expansion now has these three parameters: the coupling constant g and the two new independent and dimensional ones associated to the gluon and fermion condensates. Therefore, the invariance properties for a given quantity should be obeyed in each order of a triple Taylor expansion in those constants. The validity of the α -independence will be examined for the expansion of some quantities up to the second order in g and any order in the other two constants. In the first place the transversality of the gluon self-energy, which is one of the basic WTS identities, will be shown to be satisfied by the gluon self-energy in the above-mentioned

orders. Afterwards, in the same approximation, the dimensionally regularized effective action will also be shown to be gauge-parameter independent. In what follows, the wavy gluon and straight quark lines appearing in the Feynman diagrams without any addition will mean the complete free propagators, including the Feynman as well as the condensate components. The same lines including a central empty circle will indicate the Feynman part of the propagators (incorporating its α -dependence), and the lines showing a central black dot will represent the condensate parts. A wavy line including a central transversal cutting segment will mean the usual gluon propagator in the Feynman gauge, having a $g_{\mu\nu}$ Lorentz tensor structure. Finally, a wavy line, but including a central collinear segment, will represent the “longitudinal” part of the gluon propagator, showing the Lorentz tensor structure $(1-\alpha)p_\mu p_\nu$. This term includes the whole α -dependence of the free propagators.

5.1 Transversality of $\Pi_{\mu\nu}$ in order g^2

Figure 1 shows the diagrams contributing to the polarization operator up to the second order in g . Therefore, collecting the terms in these expressions having a certain definite order in each of the two condensate parameters will define the expansion coefficients of the polarization tensor in the series of the three parameters. All the terms of non-vanishing order in the condensate parameters are associated to the last four diagrams in Fig. 1.

The first four diagrams in this figure represent the usual second-order contribution to $\Pi_{\mu\nu}$, which is known to be transversal. The condensate-dependent term can also be represented as in Fig. 2.

Note the absence of terms coming from the quark condensate. This is a consequence of the vanishing of these terms, which is directly due to the fact that a trace of an odd number of γ matrices appears in their analytic expressions. Further the last two diagrams in Fig. 2 were the ones

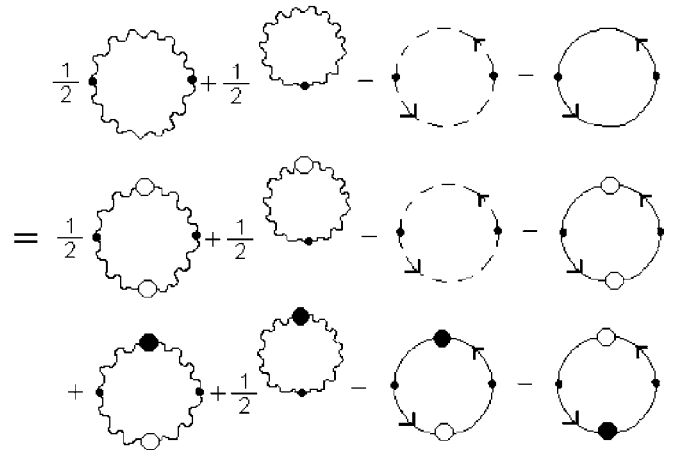


Fig. 1. The diagrams contributing to the polarization tensor up to the second order in g and all orders in the condensate parameters

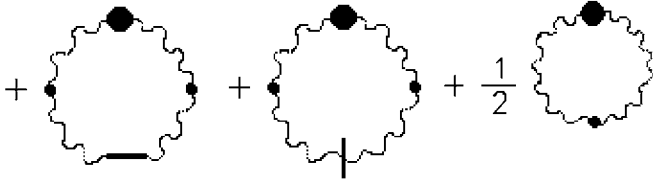


Fig. 2. Condensate dependent contributions to the polarization tensor

evaluated in [4] in the Feynman gauge and whose result is transversal. Finally, the analytic expression of the first diagram can be written in the form

$$T_{\mu\nu} = \frac{(1-\alpha)C_q}{(2\pi)^{D_i}} (-g^2) f^{a_1 a_2 a} f^{a_1 a_2 b} V_{\alpha\beta\mu}(0, p, -p) g^{\alpha\alpha'} \\ \times V_{\alpha'\sigma\nu}(0, p, -p) \frac{p^\beta p^\sigma}{(p^2)^2},$$

which, after employing the function defining the 3-gluon vertex [27]

$$V_{\mu_1\mu_2\mu_3}(k_1, k_2, k_3) = (k_1 - k_2)_{\mu_3} g_{\mu_1\mu_2} + (k_2 - k_3)_{\mu_1} g_{\mu_3\mu_2} \\ + (k_3 - k_1)_{\mu_2} g_{\mu_1\mu_3},$$

leads to

$$T_{\mu\nu}^{ab} = \frac{(1-\alpha)C_q g^2}{(2\pi)^{D_i}} f^{a_1 a_2 a} f^{a_1 a_2 b} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right),$$

showing the transversality of the polarization tensor up to second order in g and all orders in the gluon and quark condensate parameters. The α -independence of the effective action in the same approximation will be studied below.

5.2 α -independence of the effective action in order g^2

Figure 3 shows the terms of order 2 in the coupling constant (and any order in the other two parameters) of the effective action, evaluated at the zero value of the external fields. Expanding the gluon propagator as a sum of its Feynman gauge component plus the longitudinal and condensate parts; and the quark one as the usual Feynman propagator plus the quark–condensate part, the effective action can be expressed as shown in Fig. 3.

Observing this figure, it can be noted that the whole α -dependent contribution is given by the first four diagrams. However, the first two of them vanish because the longitudinal propagator is contracted with the polarization tensor in the g^2 approximation, which is transverse. Further, the third term also vanishes thanks to the fact that the longitudinal propagator is contracted with the particular self-energy part, which was shown, in the last subsection, to be transverse by itself. Finally, the self-energy quark term in the fourth diagram identically vanishes, because it contains a trace of an odd number of γ matrices. This completes the proof of the gauge invariance of the effective action in second order in the coupling constant and any order of the condensate parameters.

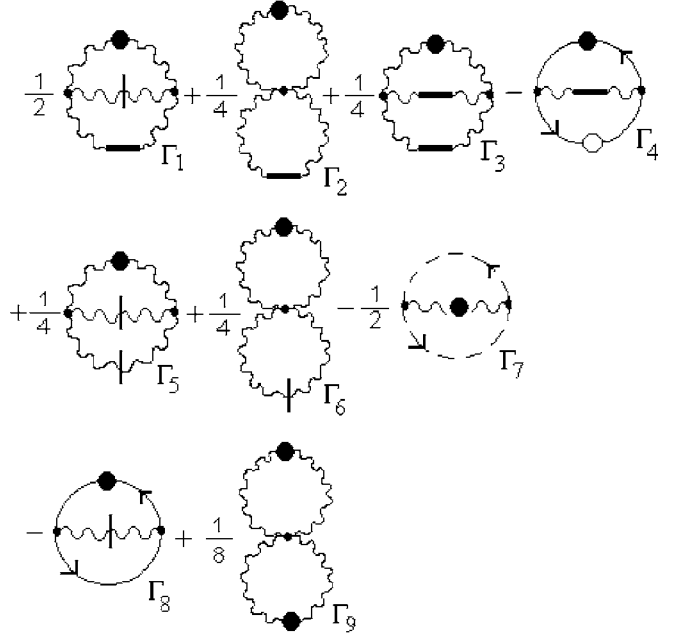


Fig. 3. Diagrams representing the contributions to the effective action in the second order in g and all orders in the gluon and quark condensates

In concluding this section it can be remarked that since massless QCD has no initial dimensional parameter, the removal of the dimensional regularization in the diagrams of the theory can be done only after partial summations are performed. These summations, then, will input the new dimensional parameters associated to the condensates, implementing in this way the dimensional transmutation effect [32]. This and other issues are expected to be addressed in our next work.

6 Summary

The gauge-invariance and regularization properties of the perturbative expansion for QCD considered in [2–4, 6] was investigated. It follows that the singularities produced by the Dirac delta-function form of the new terms added to the gluon and quark propagators can be properly eliminated by combining dimensional regularization [26] with the infrared regularization procedure for the operator quantization of gauge theories [22]. The dimensional extension allows one to cancel the singular diagrams in which factors $\delta(0)$ appear due to the momentum conservation laws at vertices, when all lines arriving are associated to condensate δ -like contributions. Further, the Nakanishi infrared procedure allows one to get rid of the singularities in the form of a Feynman propagator evaluated at zero momentum.

Those factors appear when $n - 1$ lines joined to a vertex of n legs are of the condensate kind and the remaining one is a Feynman propagator. In connection with the gauge-invariance properties, it is argued that the modi-

fied expansion should satisfy the same WTS identities as the usual PQCD. In addition, it follows that the formal functional integral representation of the generating functional of the Green functions only differs from the one associated to PQCD in the boundary conditions for the fields at $t = \pm\infty$. The formal functional integral results are checked in the second order in the coupling constant and any order in the condensate parameters. Firstly, the transversality of the gluon self-energy in the mentioned orders is shown. Afterwards, the contributions to the effective action evaluated at vanishing values of the mean fields are shown to be gauge-parameter independent. This work is expected to be extended in various directions. One of them is to implement the partial summations of the diagrams, the possibility of which was advanced in the text. This is needed in order to make the dimensional transmutation effective, thus allowing one to perform a calculation depending on the new dimensional condensate parameters. Another activity to be considered will be to specify the renormalization procedure, which in the present problem involves three parameters. Finally, a task of direct physical interest will be to incorporate knowledge of the invariance properties gained in the present work. It could bring about a better understanding of the results for the effective potential obtained in [33], in the Feynman gauge $\alpha = 1$. The effective potential calculated in this work gave signals of a strong instability of massless QCD, under the generation of a quark condensate. But it was precisely the possibility of this outcome that was the main motivation for introducing the quark condensate in the initial free vacuum in [4, 6]. Therefore, the check of the re-appearance of the instability result within a gauge invariant calculation will give further support to the dynamical generation of quark masses and condensates within massless QCD.

Acknowledgements. One of the authors (A.C.) would like to gratefully acknowledge the invitation to visit the TH Division of CERN, where this work was completed. The helpful remarks and conversations there with many colleagues such as F. Morales-Morera, S. Penaranda, R. Russo, M. Luscher, G. Altarelli, M. Mangano, R. Fleischer, K. Rummukainen, J. Bernabeu, R.K. Ellis, A. Martin, G. Veneziano, P. Minkowski, S. Pokorski, J. Ellis, G. Corcella are also greatly appreciated. Further, the support of the ICTP for the travels and the scientific activity related with the work is also acknowledged. We are also very grateful to our colleagues Danny Martínez-Pedreira and Augusto González for their comments. Finally, the friendly and efficient support of the Secretariat of the TH Division for one of the authors (A.C.) and the kind help of Suzy Vascotto in improving the English of the manuscript are very much appreciated.

Appendix A: Kugo–Ojima quantization procedure for QCD

The main elements of the Kugo–Ojima operator quantization of the free massless QCD for an arbitrary gauge parameter α will be reviewed in this appendix. The formulae

will be used in the construction of the modified perturbative expansion done in Sect. 2. The classical action for the interacting fields in the Kugo–Ojima analysis has the form

$$S = \int dx (\mathcal{L}_g + \mathcal{L}_{gh} + \mathcal{L}_B + \mathcal{L}_q) ,$$

$$\mathcal{L}_g = -\frac{1}{4} F_{\mu\nu}^a(x) F^{a\mu\nu}(x) , \quad \mathcal{L}_{gh} = -i\partial^\mu c^a(x) D_\mu^{ab}(x) c^b(x) ,$$

$$\mathcal{L}_q = \bar{\psi}^r(x) (i\gamma^\mu D_\mu^{rs}(x) - m\delta^{rs}) \psi^s(x) ,$$

$$F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c ,$$

$$D_\mu^{ab}(x) = \delta^{ab} \partial_\mu - g f^{abc} A_\mu^c , \quad D_\mu^{rs}(x) = \delta^{rs} \partial_\mu - g T_c^{rs} A_\mu^c ,$$

with

$$[T_a, T_b] = i f^{abc} T_c .$$

The equations of motion for the gluon, quarks, ghosts and auxiliary B fields for a gauge parameter α in the Kugo–Ojima quantization scheme for free massless QCD can be written as [19, 20]

$$\begin{aligned} \partial^2 A_\mu^a(x) - (1-\alpha) \partial_\mu B^a(x) &= 0 , \\ \partial^\mu A_\mu^a(x) + \alpha B^a(x) &= 0 , \\ \partial^2 B^a(x) = \partial^2 c^a(x) = \partial^2 \bar{c}^a(x) &= (i\gamma^\mu \partial_\mu - m)\psi(x) \\ &= 0 , \end{aligned} \quad (\text{A.1})$$

where, for the moment, quark fields are also considered as having an auxiliary small mass m . The notation for the Lorentz indices and field quantities will follow the one used in [27]. In the case of a general value of α , the solution for the gluon field operator is the only one that differs from its counterpart in the Feynman gauge $\alpha = 1$, which was considered in [3]. As derived in [19, 20], the non-vanishing commutation relations among the fields have the forms

$$\begin{aligned} [A_\mu^a(x), A_\nu^b(y)] &= \delta^{ab} [-g_{\mu\nu} D(x-y|0) \\ &\quad + (1-\alpha) \partial_\mu \partial_\nu E(x-y)] , \\ [A_\mu^a(x), B^b(y)] &= \delta^{ab} [-\partial_\mu D(x-y|0)] , \\ \{c^a(x), \bar{c}^b(y)\} &= i\delta^{ab} D(x-y|0) , \\ \{\psi(x), \bar{\psi}(y)\} &= (i\gamma^\mu \partial_\mu + m) D(x-y|0) , \\ D(z|m) &= \frac{1}{(2\pi)^3} \int dk s(k_0) \delta(k^2 - m^2) \exp(-ikz) , \\ E(z) &= \frac{1}{(2\pi)^3} \int dk s(k_0) \delta'(k^2) \exp(-ikz) , \end{aligned} \quad (\text{A.2})$$

with

$$s(x) = \theta(x) - \theta(-x) .$$

The invariant function E appears in the commutation relations for the gluon field when the gauge parameter differs from the Feynman gauge value $\alpha = 1$, since not all the gluon fields satisfy the d'Alembert equation. More generally, functions $E_{(\cdot)}$ are associated to each of the invariant

functions D, D_+, D_- and D_F according to

$$\begin{aligned} E_{(\cdot)}(z) &= - \lim_{m \rightarrow 0} \frac{\partial}{\partial m^2} D_{(\cdot)}(z | m) \\ &= \frac{1}{2} \frac{1}{\nabla_z^2} \left(z_0 \frac{\partial}{\partial z_0} - 1 \right) \lim_{m \rightarrow 0} D_{(\cdot)}(z | m), \end{aligned} \quad (\text{A.3})$$

where (\cdot) means any of the subindices of the functions D, D_+, D_- or D_F .

The gluon and Nakanishi B -field operators solving the above equations of motion are given as [19, 20]

$$\begin{aligned} A_\mu^a(x) &= \sum_{\mathbf{k}} \left(\sum_{\lambda=1,2} A_{\mathbf{k},\lambda}^a f_{\mathbf{k},\mu}^\lambda(x) + A_{\mathbf{k}}^{L,a} f_{\mathbf{k},L,\mu}(x) \right) \\ &\quad + B_{\mathbf{k}}^a \left[f_{\mathbf{k},S,\mu}(x) + (1-\alpha) \partial_\mu D_{(x)}^{(1/2)} g_{\mathbf{k}}(x) \right] + \text{h.c.}, \\ B^a(x) &= \sum_{\mathbf{k}} B_{\mathbf{k}}^a g_{\mathbf{k}}(x) + \text{h.c.}, \quad c^a(x) = \sum_{\mathbf{k}} c_{\mathbf{k}}^a g_{\mathbf{k}}(x) + \text{h.c.}, \\ \bar{c}^a(x) &= \sum_{\mathbf{k}} \bar{c}_{\mathbf{k}}^a g_{\mathbf{k}}(x) + \text{h.c.}, \end{aligned} \quad (\text{A.4})$$

$$\psi(x) = \sum_{\mathbf{k},s} \frac{1}{\sqrt{2V}} \left[a_{\mathbf{k}}^s u_{\mathbf{k}}^s \exp(-ikx) + b_{\mathbf{k}}^{s+} v_{\mathbf{k}}^s \exp(-ikx) \right],$$

$$\bar{\psi}(x) = \sum_{\mathbf{k},s} \frac{1}{\sqrt{2V}} \left[b_{\mathbf{k}}^s \bar{v}_{\mathbf{k}}^s \exp(ikx) + a_{\mathbf{k}}^{s+} \bar{u}_{\mathbf{k}}^s \exp(-ikx) \right],$$

where h.c. means the Hermitian conjugate of the previous term. The wave packets g and f , the polarization vectors $\varepsilon_\mu^\sigma(k)$, $\varepsilon_{L,\mu}(k)$, $\varepsilon_{S,\mu}(k)$, the Dirac spinor u and v , and the integro-differential operator $D_{(x)}^{(1/2)}$ appearing in (A.4) are defined by [19, 20]

$$g_{\mathbf{k}}(x) = \frac{1}{\sqrt{2V}k_0} \exp(-ikx),$$

$$f_{\mathbf{k},\mu}^\lambda(x) = \frac{1}{\sqrt{2V}k_0} \varepsilon_\mu^\lambda(k) \exp(-ikx),$$

where $kx = k_0 x_0 - \mathbf{k} \cdot \mathbf{x}$, $k_0 = |\mathbf{k}|$,

$$\mathbf{k} \cdot \varepsilon^\lambda(k) = 0, \quad \varepsilon_0^\lambda(k) = 0, \quad \varepsilon^\lambda(k) \cdot \varepsilon^\tau(k) = \delta^{\lambda\tau},$$

for $\lambda, \tau = 1, 2$,

$$\varepsilon_{L,\mu}(k) = -i k_\mu = -i(|\mathbf{k}|, -\mathbf{k}),$$

$$\varepsilon_{S,\mu}(k) = -i \frac{\bar{k}_\mu}{2|\mathbf{k}|^2} = \frac{-i(|\mathbf{k}|, \mathbf{k})}{2|\mathbf{k}|^2},$$

$$D_{(x)}^{(1/2)} \equiv \frac{1}{2} (\nabla^2)^{-1} (x_0 \partial_0 - 1/2),$$

$$u_{\mathbf{k}}^s \equiv u^s(k) = \sqrt{m+k_0} \begin{pmatrix} u^s(0) \\ \frac{\sigma \mathbf{k}}{m+k_0} u^s(0) \end{pmatrix},$$

$$v_{\mathbf{k}}^s \equiv v^s(k) = \sqrt{m+k_0} \begin{pmatrix} \frac{\sigma \mathbf{k}}{m+k_0} v^s(0) \\ v^s(0) \end{pmatrix}^\top. \quad (\text{A.5})$$

The operator $D_{(x)}^{(1/2)}$ works as an ‘‘inverse’’ of the d’Alembertian for simple pole functions [22], that is

$$\partial^2 D_{(x)}^{(1/2)} f(x) = f(x) \quad \text{if} \quad \partial^2 f(x) = 0.$$

A large cubic box of volume $V = L^3$ in a particular Lorentz reference frame is assumed for the imposition of periodic boundary conditions on the fields. Accordingly, the spatial momenta in the above sums take the values $\mathbf{k} = \frac{2\pi}{L}(n_1, n_2, n_3)$, for arbitrary integers n_1, n_2 and n_3 . The four-vectors for all the particles but the quarks are null ones, $k = (|\mathbf{k}|, \mathbf{k})$, and for the quarks $k = (\sqrt{|\mathbf{k}|^2 + m^2}, \mathbf{k})$. The matrix below shows the commutation relations between the creation and annihilation operators for the various fields:

$$\left\{ \begin{array}{cccccccc} & A_{\mathbf{k}',\lambda'}^{a'+} & A_{\mathbf{k}'}^{L,a'+} & B_{\mathbf{k}'}^{a'+} & c_{\mathbf{k}'}^{a'+} & \bar{c}_{\mathbf{k}'}^{a'+} & a_{\mathbf{k}'}^{r'+} & b_{\mathbf{k}'}^{r'+} \\ A_{\mathbf{k},\lambda}^a & \delta^{aa'} \delta_{\mathbf{k}\mathbf{k}'} \delta_{\lambda\lambda'} & 0 & 0 & 0 & 0 & 0 & 0 \\ A_{\mathbf{k}}^{L,a} & 0 & 0 & -\delta^{aa'} \delta_{\mathbf{k}\mathbf{k}'} & 0 & 0 & 0 & 0 \\ B_{\mathbf{k}}^a & 0 & -\delta^{aa'} \delta_{\mathbf{k}\mathbf{k}'} & 0 & 0 & 0 & 0 & 0 \\ c_{\mathbf{k}}^a & 0 & 0 & 0 & 0 & i\delta^{aa'} \delta_{\mathbf{k}\mathbf{k}'} & 0 & 0 \\ \bar{c}_{\mathbf{k}}^a & 0 & 0 & 0 & -i\delta^{aa'} \delta_{\mathbf{k}\mathbf{k}'} & 0 & 0 & 0 \\ a_{\mathbf{k}}^r & 0 & 0 & 0 & 0 & 0 & \delta^{rr'} \delta_{\mathbf{k}\mathbf{k}'} & 0 \\ b_{\mathbf{k}}^r & 0 & 0 & 0 & 0 & 0 & 0 & \delta^{rr'} \delta_{\mathbf{k}\mathbf{k}'} \end{array} \right\}. \quad (\text{A.6})$$

Appendix B: Longitudinal and scalar modes contribution

The longitudinal and scalar modes contribution is determined by the expression

$$\begin{aligned} &\langle 0 | \exp [C_3^* (\alpha, P) B_{\mathbf{p}_i}^a A_{\mathbf{p}_i}^{L,a} + D^* (\alpha, P) B_{\mathbf{p}_i}^a B_{\mathbf{p}_i}^a] \\ &\times \exp \left\{ i \int dx J^{\mu,a}(x) (A_{\mathbf{p}_i}^{L,a+} f_{\mathbf{p}_i,L,\mu}^*(x) + B_{\mathbf{p}_i}^{a+} [f_{\mathbf{p}_i,S,\mu}^*(x) \right. \\ &\quad \left. + (1-\alpha) (A^*(p_0, x_0) f_{\mathbf{p}_i,L,\mu}^*(x) + B_\mu^*(p_i, x))] \right\} \\ &\times \exp \left\{ i \int dx J^{\mu,a}(x) (A_{\mathbf{p}_i}^{L,a} f_{\mathbf{p}_i,L,\mu}(x) + B_{\mathbf{p}_i}^a [f_{\mathbf{p}_i,S,\mu}(x) \right. \\ &\quad \left. + (1-\alpha) (A(p_0, x_0) f_{\mathbf{p}_i,L,\mu}(x) + B_\mu(p_i, x))] \right\} \\ &\times \exp [C_3 (\alpha, P) B_{\mathbf{p}_i}^{a+} A_{\mathbf{p}_i}^{L,a+} + D (\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+}] | 0 \rangle. \end{aligned} \quad (\text{B.1})$$

As in [3, 5], we first calculate

$$\begin{aligned} &\exp \left\{ i \int dx J^{\mu,a}(x) (A_{\mathbf{p}_i}^{L,a} f_{\mathbf{p}_i,L,\mu}(x) + B_{\mathbf{p}_i}^a [f_{\mathbf{p}_i,S,\mu}(x) \right. \\ &\quad \left. + (1-\alpha) (A(p_0, x_0) f_{\mathbf{p}_i,L,\mu}(x) + B_\mu(p_i, x))] \right\} \\ &\times \exp [C_3 (\alpha, P) B_{\mathbf{p}_i}^{a+} A_{\mathbf{p}_i}^{L,a+} + D (\alpha, P) B_{\mathbf{p}_i}^{a+} B_{\mathbf{p}_i}^{a+}] | 0 \rangle \\ &= \exp \left\{ C_3 (\alpha, P) \left(B_{\mathbf{p}_i}^{a+} - i \int dx J^{\mu,a}(x) f_{\mathbf{p}_i,L,\mu}(x) \right) \right\} \end{aligned}$$

$$\begin{aligned}
& \times \left(A_{\mathbf{p}_i}^{L,a+} - i \int dx J^{\mu,a}(x) [f_{p_i,S,\mu}(x) + (1-\alpha) \right. \\
& \times (A(p_0, x_0) f_{p_i,L,\mu}(x) + B_\mu(p_i, x))] \left. \right) \\
& + D(\alpha, V) \left(B_{\mathbf{p}_i}^{a+} - i \int dx J^{\mu,a}(x) f_{p_i,L,\mu}(x) \right) \\
& \times \left(B_{\mathbf{p}_i}^{a+} - i \int dx J^{\mu,a}(x) f_{p_i,L,\mu}(x) \right) \Big\} |0\rangle. \quad (\text{B.2})
\end{aligned}$$

A similar expression is obtained acting to the left in (B.1).

Substituting the above results in (B.1), and introducing the following notation:

$$\begin{aligned}
C^* &\equiv C_3^*(\alpha, P), \quad C \equiv C_3(\alpha, P), \\
D^* &\equiv D^*(\alpha, P), \quad D \equiv D(\alpha, P), \\
\hat{A}^+ &\equiv A_{\mathbf{p}_i}^{L,a+}, \quad \hat{A} \equiv A_{\mathbf{p}_i}^{L,a}, \quad \hat{B}^+ \equiv B_{\mathbf{p}_i}^{a+}, \quad \hat{B} \equiv B_{\mathbf{p}_i}^a, \\
a_1 &\equiv -i \int dx J^{\mu,a}(x) [f_{p_i,S,\mu}^*(x) + (1-\alpha)(A^*(p_0, x_0) \\
& \times f_{p_i,L,\mu}^*(x) + B_\mu^*(p_i, x))], \\
a_2 &\equiv -i \int dx J^{\mu,a}(x) [f_{p_i,S,\mu}(x) + (1-\alpha)(A(p_0, x_0) \\
& \times f_{p_i,L,\mu}(x) + B_\mu(p_i, x))], \\
b_1 &\equiv -i \int dx J^{\mu,a}(x) f_{p_i,L,\mu}^*(x), \\
b_2 &\equiv -i \int dx J^{\mu,a}(x) f_{p_i,L,\mu}(x), \quad (\text{B.3})
\end{aligned}$$

one obtains

$$\begin{aligned}
& \langle 0 | \exp \left\{ C^* (\hat{A} + a_1) (\hat{B} + b_1) + D^* (\hat{B} + b_1) (\hat{B} + b_1) \right\} \\
& \times \exp \left\{ C (\hat{B}^+ + b_2) (\hat{A}^+ + a_2) \right. \\
& \left. + D (\hat{B}^+ + b_2) (\hat{B}^+ + b_2) \right\} |0\rangle. \quad (\text{B.4})
\end{aligned}$$

Following exactly the same procedure previously described in [3, 5], a recurrence relation is obtained for (B.4):

$$\begin{aligned}
& \exp \left\{ C^* a_1 b_1 + D^* b_1^2 + [C(C^* a_1 - a_2) + 2CD^* b_1] \right. \\
& \times (C^* b_1 - b_2) \sum_{m=0}^n [|C|^{2(2m)} + |C|^{2(2m+1)}] + (C^* b_1 - b_2)^2 \\
& \times \left(D \sum_{m=0}^n [(2m+1)|C|^{2(2m)} + 2(m+1)|C|^{2(2m+1)}] \right. \\
& \left. + C^2 D^* \sum_{m=0}^n [2m|C|^{2(2m-1)} + (2m+1)|C|^{2(2m)}] \right) \left. \right\} \\
& \times \langle 0 | \exp \left\{ C^* \hat{A} \hat{B} + D^* \hat{B} \hat{B} \right\} \\
& \times \exp \left\{ C^{*n+1} C^{n+1} (C^* b_1 - b_2) \hat{A} \right. \\
& + [C^{*n+1} C^{n+1} (C^* a_1 - a_2) + 2C^{*n+1} C^{n+1} D^* b_1 \\
& \left. + 2C^{*n} C^n (n+1) (C^* D + CD^*) (C^* b_1 - b_2) \right] \hat{B} \left. \right\}
\end{aligned}$$

$$\times \exp \left\{ C \hat{B}^+ \hat{A}^+ + D \hat{B}^+ \hat{B}^+ \right\} |0\rangle. \quad (\text{B.5})$$

We then assume that $|C| < 1$ so that

$$\begin{aligned}
& \lim_{n \rightarrow \infty} |C|^{2n} = \lim_{n \rightarrow \infty} n |C|^{2n} = 0, \\
& \lim_{n \rightarrow \infty} \sum_{m=0}^n [|C|^{2(2m)} + |C|^{2(2m+1)}] = \frac{1}{(1-|C|^2)}, \\
& \lim_{n \rightarrow \infty} \sum_{m=0}^n [(2m+1) |C|^{2(2m)} + 2(m+1) |C|^{2(2m+1)}] \\
& = \frac{1}{(1-|C|^2)^2}, \quad (\text{B.6})
\end{aligned}$$

which allows one to rewrite (B.5) as

$$\begin{aligned}
& \exp \left\{ C^* a_1 b_1 + D^* b_1^2 + [C(C^* a_1 - a_2) + 2CD^* b_1] \right. \\
& \times \left. \frac{(C^* b_1 - b_2)}{(1-|C|^2)} + (C^* b_1 - b_2)^2 \frac{(D + C^2 D^*)}{(1-|C|^2)^2} \right\} \\
& \langle 0 | \exp \left\{ C^* \hat{A} \hat{B} + D^* \hat{B} \hat{B} \right\} \exp \left\{ C \hat{B}^+ \hat{A}^+ + D \hat{B}^+ \hat{B}^+ \right\} |0\rangle. \quad (\text{B.7})
\end{aligned}$$

Replacing the compact notation (B.3) in (B.7), and expanding all functions of \mathbf{p}_i in the vicinity of $\mathbf{p}_i = 0$ (keeping in mind that the sources are located in a finite space region), it is sufficient to consider only the first term in all expansions. After that (B.7) takes the form (26).

References

1. T. Schäfer, E.V. Shuryak, Rev. Mod. Phys. **70**, 323 (1998)
2. A. Cabo, S. Peñaranda, R. Martinez, Mod. Phys. Lett. A **10**, 2413 (1995)
3. M. Rigol, A. Cabo, Phys. Rev. D **62**, 074018 (2000) [hep-th/9909057]
4. A. Cabo, M. Rigol, Eur. Phys. J. C **23**, 289 (2002) [hep-ph/0008003]
5. M. Rigol, About an alternative vacuum state for perturbative QCD, Graduate Dissertation Thesis, Instituto Superior de Ciencias y Tecnología Nucleares, La Habana, Cuba, 1999
6. A. Cabo, JHEP **04**, 044 (2003) [hep-ph/0209215(2002)]
7. G.K. Savvidy, Phys. Lett. B **71**, 133 (1977)
8. I.A. Batalin, S.G. Matinyan, G.K. Savvidy, Sov. J. Nucl. Phys. **26**, 214 (1977)
9. A. Cabo, O.K. Kalashnikov, A.E. Shabad, Nucl. Phys. B **185**, 473 (1981)
10. W. Dittrich, M. Reuter, Phys. Lett. B **128**, 321 (1983)
11. P. Hoyer, HIP-2002-44-TH, September 2002. 3pp.; Talk given at 31st International Conference on High Energy Physics, ICHEP 367-369, Amsterdam (2002) [hep-ph/0209318]

12. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385, 448, 519 (1979)
13. R. Fukuda, Phys. Rev. D **21**, 485 (1980)
14. K.G. Chetyrkin, S. Narison, V.I. Zakharov, Nucl. Phys. B **550**, 353 (1999)
15. S.J. Huber, M. Reuter, M.G. Schmidt, Phys. Lett. B **462**, 158 (1999)
16. P. Hoyer, NORDITA - 96/63 P (1996), hep-ph/9610270 (1996); P. Hoyer, NORDITA - 97/44 P (1997), hep-ph/9709444 (1997)
17. H.J. Munczek, A.M. Nemirovsky, Phys. Rev. D **55**, 3455 (1983)
18. C.J. Burden, C.D. Roberts, A.G. Williams, Phys. Lett. B **285**, 347 (1992)
19. T. Kugo, I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979)
20. N. Nakanishi, I. Ojima, Covariant operator formalism of gauge theories and quantum gravity, (Singapore, World Scientific, 1990)
21. H.P. Pavel, D. Blaschke, V.N. Pervushin, G. Röpke, Int. J. Mod. Phys. A **14**, 205 (1999)
22. N. Nakanishi, Prog. Theor. Phys. **51**, 952 (1974); N. Nakanishi, Prog. Theor. Phys. **52**, 1929 (1974)
23. D.G. Boulware, Phys. Rev. D **23**, 389 (1981)
24. B.S. DeWitt, in Proceedings: Symposium on Quantum Gravity II, eds. C. Isham, R. Penrose, D. Sciama, (Clarendon Press, Oxford, 1981)
25. G. 't Hooft, in 12th Winter School of Theoretical Physics Functional and Probabilistic Methods in Quantum Field Theory, Karpacz, Poland, 17 February–2 March 1975, Acta Universitatis Wratislaviensis No. 368, eds. J. Lopuszanski, B. Jancewicz, Vol. 1, 1976
26. G. Leibbrandt, Rev. Mod. Phys. **47**, 849 (1975)
27. T. Muta, Foundations of Quantum Chromodynamics, World Scientific Lectures Notes in Physics - Vol. 5, 1987
28. S. Gasiorowicz, Elementary particle physics (John Wiley, New York, 1966)
29. L.D. Faddeev, A.A. Slanov, Gauge fields. Introduction to quantum theory (Benjamin Cummings Publishing, 1980)
30. B. Sakita, Quantum Theory of Many Variable Systems and Fields, World Scientific Lecture Notes in Physics, Vol. 1, 1985
31. J. Zinn-Justin, in Trends in Elementary Particle Theory, Lecture Notes in Physics, Vol. 37, Springer, 1975
32. S. Coleman, E. Weinberg, Phys. Rev. D **7**, 1888 (1973)
33. A. Cabo, D. Martinez-Pedreira, ICTP Preprint IC/2004/118 (2004), hep-ph/0501054(2005); to appear in Eur. Phys. J. C